Mathematics Assessment Project

Formative Assessment Lesson Materials

Using Positive and Negative Numbers in Context

MARS Shell Center
University of Nottingham & UC Berkeley
Alpha Version

Please Note:

These materials are still at the "alpha" stage and are not expected to be perfect. The revision process concentrated on addressing any major issues that came to light during the first round of school trials of these early attempts to introduce this style of lesson to US classrooms. In many cases, there have been very substantial changes from the first drafts and new, untried, material has been added.

We suggest that you check with the Nottingham team before releasing any of this material outside of the core project team.

If you encounter errors or other issues in this version, please send details to the MAP team c/o map.feedback@mathshell.org.

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Using Positive and Negative Numbers in Context

Mathematical goals

This lesson unit is intended to help you assess how well students are able to understand and use directed numbers in context, and, in particular, to help you identify and aid students who have difficulties in ordering, comparing, adding, and subtracting positive and negative integers. Particular attention is paid to the use of negative numbers on number lines to explore the structures:

starting temperature + change in temperature = final temperature.
final temperature – change in temperature = starting temperature.
final temperature – starting temperature = change in temperature

Common Core State Standards

This lesson involves *mathematical content* in the standards from across the grades, with emphasis on aspects of:

7.NS Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.
7.EE Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

This lesson involves a range of *mathematical practices,* with emphasis on:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
4. Model with mathematics.
7. Look for and make use of structure.

Introduction

The lesson unit is structured in the following way:

- Before the lesson, each student works individually on an assessment task designed to help you understand their current levels of understanding and difficulties. You then review their responses and write questions for students to answer that should help them improve their work.
- After a whole-class introduction, students work together collaboratively in small groups on a card matching task. In the same small groups, students then create directed number calculations to correspond to temperature changes or city temperatures. To end the lesson there is a whole-class discussion.
- In a follow-up session, students work alone again on a similar task to the introductory task.

Materials required

- Each individual student will need a copy of the assessment task, *Temperature Changes*, a copy of the review task, *Temperature Changes (Again),* a copy of the *Temperature Scale*, a mini-whiteboard, a pen, and an eraser.
- Each small group of students will need a set of cut-up cards from the sheets *City Temperatures* and *Changes in Temperature*, a glue stick, and a large sheet of paper for making a poster.
- You will also need one copy of the cut-up cards *Introduction to Negative Numbers*.
- There are some projectable resources to help with discussions.

Time needed

Approximately 20 minutes before the lesson, a one-hour lesson, and 20 minutes in a follow-up lesson (or for homework). Timings given are only approximate. Exact timings will depend on the needs of the class.

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Before the lesson

Pre-Assessment task: *Temperature Changes* (15 minutes)

Have students complete this task in class or for homework a few days before the formative assessment lesson. This will give you an opportunity to assess the work, and to find out the kinds of difficulties students have with it You will then be able to target your help more effectively in the follow-up lesson.

Give each student a copy of the task *Temperature Changes*.

> *Spend fifteen minutes on your own, answering these questions.*
>
> *Explain all your answers carefully.*

It is important that students are allowed to answer the questions without your assistance, as far as possible.

Students should not worry too much if they cannot understand or do everything, because in the next lesson they will engage in a similar task, which should help them to progress.

Temperature Changes

1.

The temperature was +3°C at midday.
By evening, the temperature was −5°C.

(a) Some of the calculations below show how to figure out the temperature change.

Circle any that apply.

3 − 5	3 + (−5)	(−5) − 3	(−5) + 3

(b) What was the change in temperature? Explain your answer.

.....

.....

2.

The temperature was −7°C at midnight.
By the next day, the temperature had risen by 11°C.

(a) Some of these calculations show how to figure out the temperature the next day.

Circle any that apply.

7 − 11	11 + (−7)	(−11) − (−7)	(−7) + 11

(b) What was the temperature the next day? Explain your answer.

.....

.....

.....

3. Here is a calculation:

5 − (−11)

(a) Figure out the answer to the calculation:

.....

.....

(b) Which of these story questions fits the calculation? Circle any that apply.

A	It was really cold at midnight. During the morning, the temperature rose 5°C. By midday, it had reached -11°C. What was the temperature at midnight?
B	The temperature at midnight was −11°C. By midday, the temperature was 5°C. What was the temperature change?
C	At midday, the temperature was 5°C. The temperature then fell by 11°C. What was the final temperature?

Explain your answer:

.....

Assessing students' responses

Collect students' responses to the task, and note what their work reveals about their current levels of understanding and their different approaches.

We suggest that you do not score students' work. The research shows that this will be counterproductive, as it will encourage students to compare their scores, and will distract their attention from what they can do to improve their mathematics. Instead, help students to make further progress by summarizing their difficulties as a series of questions. Some suggestions for these are given on the next page. These have been drawn from common difficulties observed in trials of this unit.

We suggest you select questions that apply to your students' work. At the bottom of the table, add your own questions, based on your students' solutions. You may choose to write questions on each student's work. If you do not have time to do this, select a few questions that will be of help to the majority of students. These can be written on the board at the start of the follow-up lesson.

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Common issues:	Suggested questions and prompts:
Student chooses an expression that does not represent the story For example: The student chooses the 1st, 2nd or 4th expressions to represent the temperature change (Q1). Or: The student chooses the 1st or 3rd expressions to represent the next-day temperature (Q2).	• *Write what you are asked to figure out in words. How can you write this calculation in math?* • *Mark the start and finish temperatures on your Temperature Scale. Does the temperature rise or fall? What is the difference between the two temperatures? Is the temperature change positive or negative?* • *What does the negative sign represent in −7? What does the middle negative sign represent in (−11) − (−7)?*
Student makes a calculation error For example: The student writes "−5 − 3 = − 2". Or: The student writes "(−5) + (−11) = (−6).	• *Mark the beginning and end temperatures on your Temperature Scale. From the first figure, which direction do you move in? Does your solution make sense?* • *What math did you use to arrive at your solution?*
Student chooses a story that is not represented by the expression (Q3) For example: The student connects the expression (−5) + (−11) with Story A or Story B.	• *How does the calculation represent what happens to the temperature in the story? How can you write that in words?* • *(Story A) What is the temperature at 8am? Mark this on your Temperature Scale. By how much does the temperature change to reach that temperature? Write this story as a calculation.* • *(Story B) What was the temperature at midnight? What was the temperature at midday? Mark these on your temperature scale. By how much does the temperature change? How would you write this story as a calculation?*
Student provides little or no explanation for her answers For example: The student writes down correct numerical solutions but does not provide any explanation of how he/she generated them. Or: The student writes correct procedures but does not explain the links between context and operations.	• *Imagine students in a lower Grade are going to read your solution. How can you explain very clearly, so that they understand?* • *How do you know that this calculation matches that story? Explain your answer clearly.*
Student answers all questions correctly	• *Make up a calculation using positive and negative numbers. Now write a story to fit that calculation. Explain how the calculation and story fit together.*

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Suggested lesson outline

Whole-class interactive introduction (10 minutes)

Give each student a mini-whiteboard, a pen, and an eraser.

Explain to the class that in the lesson they will be working with positive and negative numbers in the context of temperatures, in degrees Celsius.

In degrees Celsius, at what temperature does water boil? [100°C]

In degrees Celsius, at what temperature does water freeze? [0°C]

In degrees Celsius, what is an average temperature for a summer's/winter's day? [Depends on location]

Give each student one copy of *Temperature Scales*.

Ask four students to come to the front of the room and give each of them a card from the sheet *Card Set: Introduction to Negative Numbers*.

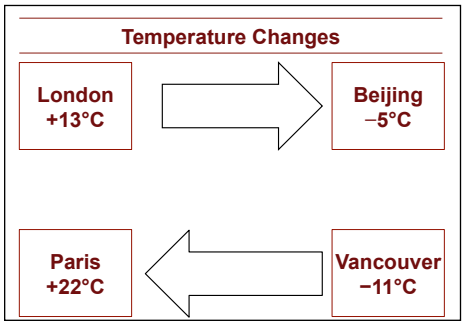
Ask the four students to stand in order of temperature from coldest to hottest.

Who thinks they are in the correct order? Who thinks they are not in the correct order?

If students disagree with the order, ask one or two to explain their thinking. Many students find it difficult to come to terms with the notion that –11°C is colder than –8° C.

Ask the students to return to their seats.

Now project *Slide 1* of the projector resource onto the board (or draw the diagrams from the slide on the board).



Ask students to write on their mini-whiteboards the answers to questions such as the following:

If I fly from London to Beijing, does the temperature rise or fall?
What is the change in temperature? [–18°C.]

If I fly from Vancouver to Paris, does the temperature rise or fall?
What is the change in temperature? [+33°C.]

The temperature change is +24°C. What was my journey? [Vancouver to London.]

The temperature change is –6°C. What was my journey? [Beijing to Vancouver.]

Cairo is 30°C warmer than Beijing. What is the temperature in Cairo? [+25°C.]

After each question, ask students to explain how they figured out their answer. Encourage the rest of the class to challenge their reasoning.

Write on the board the temperature changes between the relevant cities. These figures should go in the arrows or in newly drawn arrows.

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Collaborative work 1 (10 minutes)

Organize the class into groups of two or three students.

Give each group a large sheet of paper for making a poster, the *Card Set: City Temperatures*, and *Card Set: Changes in Temperature*.

> *What do you think the two sets of cards represent?*

Explain to students how they should work together:

> *Take turns to place cards.*
>
> *When it is your turn, connect two City Temperature cards using a Changes in Temperature arrow card. Figure out any missing temperatures.*
>
> *Give reasons for your calculations.*
>
> *If others in your group disagree, they should explain why. Then figure out the answer together.*
>
> *When you have reached an agreement, write your temperature in the box on the card.*

To remind students about these instructions, you could use *Slide 2*, *Instructions for Working Together*.

While students are working, you have two tasks: to find out about students' work, and to support their reasoning.

Find out about students' work

As you move around the room listen to students' explanations. Are they able to interpret the operations in terms of this context? Notice the vocabulary that they use to describe operations on numbers in the context. Do they find it more difficult to find a missing temperature, or the change in temperature? Do they make any errors, either in the choice of calculation, or in their arithmetic?

Note any difficulties that emerge for more than one group; these can be discussed later in the lesson.

Support student reasoning

If you hear students providing incorrect explanations, try not to make suggestions that move students towards a particular approach to the task. Instead, ask questions that help to identify their errors and to redirect their thinking.

> *Jordan wrote this temperature on this card. Della, do you think Jordan is correct? Why?*
>
> *Is it hotter in [Beijing or Vancouver]? Does this mean the number on the Changes in Temperature card will be positive or negative?*
>
> *What does a negative temperature on the Changes in Temperature card mean?*
>
> *How can you check you've calculated the correct temperature for this city? [Use a different route to get to the city.]*

You may want to also use the questions in the *Common issues* table to support your own questioning.

If students are struggling, you might refer a group of students to another group who have dealt with an issue well. If several groups of students are finding the same issue difficult, you might write a suitable question on the board, or organize a brief whole-class discussion focusing on that aspect of mathematics.

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Sharing posters (10 minutes)

Give each small group a glue stick.

As students finish matching the cards, ask students to share their solutions with a neighboring group.

Check to see if the figures on any of the cards are different from your own.

Someone in each group needs to explain the reasoning behind these figures. If anything is unclear, ask for clarification.

Then all working together, consider if you should change any of your answers.

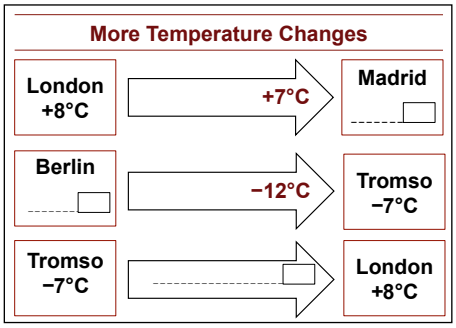
It is important that everyone in both groups understands the math. You are responsible for each other's learning.

When students are satisfied with their cards, they are to glue them onto the poster.

Whole-class discussion (10 minutes)

This activity allows students to write the different temperatures as an arithmetic sum.

Project or draw *Slide 3* of the projector resource onto the board.



Ask each of the following questions in turn. If you think your students will struggle to write the arithmetic, you could first ask them to describe in words how they calculated each answer.

What is the temperature in Madrid? [15 °C.]

Write this answer in the box below 'Madrid'.

What is the sum for calculating this answer? [+8 + (+7) = +15.]

Write the calculation along the dashed line below 'Madrid'.

What is the temperature in Berlin? [5 °C.]

Write the answer in the box below 'Berlin'.

What is the arithmetic for calculating this answer?

Students may use different strategies to figure out the answer. Some may answer the question by working in reverse, that is, finding the temperature in Berlin by flying from Tromso to Berlin: –7 + 12 = + 5. Others may answer: –7 – (–12) = +5. Write both answers along the dashed line below 'Berlin'.

What is the temperature change when you fly from Tromso to London? [15 °C]

Write this answer in the box in the third arrow.

What is the arithmetic for calculating this answer? [+8 – (–7) = +15.]

Write the calculation along the dashed line in the third arrow.

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You may also want to ask:

> *Suppose you don't know the final temperature. For example, you didn't know the temperature in Madrid. What is the general method used to figure out a missing final temperature? [Starting temperature + Change = Final temperature.]*

> *Suppose you don't know the starting temperature. For example, you didn't know the temperature in Berlin. What is the general method used to figure it out? [Final temperature – Change = Starting temperature.]*

> *Suppose you don't know the temperature change, for example, when flying from Tromso to London. What is the general method used to figure it out? [Final temperature – Starting temperature = Change in temperature.]*

To make it easier for your students to figure out the general formulas, you may choose to only use the journey between London and Madrid.

Collaborative work 2 (10 minutes)

Now ask students to write a calculation for each of the temperatures they have already written in the boxes on either the *City Temperature* cards or the *Changes in Temperature* cards. These sums should be written on the dashed lines.

Support the students as you did in the previous collaborative activity. As students finish the task, ask them to share their solutions with a neighboring group, as they did before.

Whole-class discussion (10 minutes)

Organize a discussion about what has been learned. Try to focus the discussion on any common errors and misconceptions you noticed in the collaborative work. You may want to draw on the questions in the *Common issues* table to support your own questioning. The intention is that students develop their understanding by talking about and correcting each other's errors. Try to resist simply explaining the mistakes students have made yourself.

Ask each pair of students to report back on one of their calculations. Concentrate on the arithmetic students found difficult.

> *Matt, how did you calculate this Temperature/Temperature Change?*

> *Mary, do you have a different method for obtaining the same Temperature/Temperature Change?*

You may also want to investigate combining changes:

> *If I fly from Detroit to Anchorage, does the temperature rise or fall? By how much? [–20°C.] Write a calculation for this answer. Write a different calculation.*

> *How can you calculate the fall in temperature? [15 – 35 = –20°C]*

> *Make up a journey you've not already looked at. Tell me about the temperatures. Tell me the temperature change.*

Ask students to try to generalize rules for adding or subtracting positive and negative quantities from these or similar examples.

> *What is the effect of adding/subtracting a positive/negative number? Give me an example.*

Follow-up lesson: Reviewing the assessment task (10 minutes)

Give each student a copy of the review task, *Temperature Changes (Again),* and their original scripts from the assessment task, *Temperature Changes*. If you have not added questions to individual pieces of work, then write your list of questions on the board. Students should select from this list only the questions they think are appropriate to their own work.

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Read through your scripts from Temperature Changes, and the questions [on the board/written on your script]. Think about what you have learned this lesson.

Now look at the new task sheet, Temperature Changes (Again). Can you use what you've learned to answer these questions?

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Solutions

Pre-Assessment task: *Temperature Changes*

The assessment is designed to check whether students can make sense of simple arithmetic operations on negative numbers in context. This is partly about being able to deal with ambiguity in English language descriptions of operations.

1. (a) The calculations 3 − 5, 3 + (−5), and (−5) + 3 all give the answer −2. Only the third calculation, (−5) − 3 = − 8, fits the story, indicating by how much the temperature has changed. This uses the structure: final temperature – initial temperature = change in temperature.

   (b) When reading a student's chosen method, notice whether they just write a number, use a jotted strategy such as writing how they counted down, or use a written method.

2. (a) 7 − 11 = −4, (−11) − (−7) = − 4 are incorrect. The calculations 11 + (−7) = 4 and (−7) + 11= 4 show how to find the final temperature by adding the starting temperature to the temperature change.

   (b) Again, notice whether the student just writes a number, uses a jotted strategy such as writing how to count up from −7, or uses a written method.

3. (a) 5 − (−11) = +16. Noticing the method of calculation is as important as checking the correct numerical solution.

   (b) *Story B* fits the calculation: final temperature – initial temperature = change in temperature.

   Story A gives the change in temperature and the final temperature. To calculate the initial temperature you would require (−11) – 5 = −16 (or an equivalent calculation).

   Story C gives the initial temperature and the change in temperature. To find the final temperature, you would need to calculate 5+ (−11).

Post-Assessment task: *Temperature Changes (Again)*

1. a) The second and third calculations could both be used to calculate how much the temperature has changed. The second calculation suggests that the student is using the idea of subtracting the initial temperature from the final temperature. The third calculation might suggest that the student is using a number line, moving to zero then finding the size of the negative component.

   (b) When reading a student's chosen method, notice whether they just write a number, use a jotted strategy such as writing how they count down, or use a written method.

2. (a) 9 + (−2)= 7 and (−2) + 9 are both ways of figuring out the final temperature.

   (b) Again, notice whether the student just writes a number, uses a jotted strategy, or uses a written method.

3. (a) 9 − (−4) = 13. Noticing the method of calculation is as important as checking the correct numerical solution.

   (b) *Story B* fits the calculation: by subtracting the initial temperature from the final temperature, you figure out the change in temperature: 9 − (− 4) = 13.

   Story A gives the change in temperature and the final temperature. To calculate the start temperature you would add the negative change in temperature to the initial temperature: 9 + (− 4) or 9 − 4.

   Story C gives the final temperature and change in temperature. To find the initial temperature, you subtract the change from the final temperature. That calculation would be −4 – 9 = −13

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Lesson Task

Students may use different routes to figure out the temperatures.

Detroit **−5 + (+15) = 10°C**	New York to Detroit **+15°C**	New York **−5°C**
Detroit to Honolulu **+25 − (+10) = +15°C**	**Honolulu to New York** −5 − (+25) = −30°C	Anchorage to New York **−5 − (−10) = +5°C**
Honolulu **+25°C**	Honolulu to Anchorage −35°C	Anchorage **+25 +(−35) = −10°C**
Fairbanks to Honolulu +50°C	**Fairbanks to Anchorage** −10 − (−25) = +15°C	Anchorage to Washington +30°C
Fairbanks **+20 + (−45) =−25°C**		Washington **−10+ (+30) = 20°C**
Las Vegas to Fairbanks −45°C	Washingto to Las Vegas **+20 − (+20) = 0**	Washington to New Orleans −10°C
Las Vegas **+20°C**	New Orleans to Las Vegas **+20 − (+10) = +10°C**	New Orleans **+10°C**

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Temperature Changes

1.

The temperature was +3°C at midday.

By evening, the temperature was −5°C.

(a) Some of the calculations below show how to figure out the temperature change.

Circle any that apply.

3 − 5	3 + (−5)	(−5) − 3	(−5) + 3

(b) What was the change in temperature? Explain your answer.

2.

The temperature was −7°C at midnight.

By the next day, the temperature had risen by 11°C.

(a) Some of these calculations show how to figure out the temperature the next day.

Circle any that apply.

7 − 11	11 + (−7)	(−11) − (−7)	(−7) + 11

(b) What was the temperature the next day? Explain your answer.

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3. Here is a calculation:

$$5 - (-11)$$

(a) Figure out the answer to the calculation:

(b) Which of these story questions fits the calculation? Circle any that apply.

A	It was really cold at midnight. During the morning, the temperature rose 5°C. By midday, it had reached -11°C. What was the temperature at midnight?
B	The temperature at midnight was −11°C. By midday, the temperature was 5°C. What was the temperature change?
C	At midday, the temperature was 5°C. The temperature then fell by 11°C. What was the final temperature?

Explain your answer:

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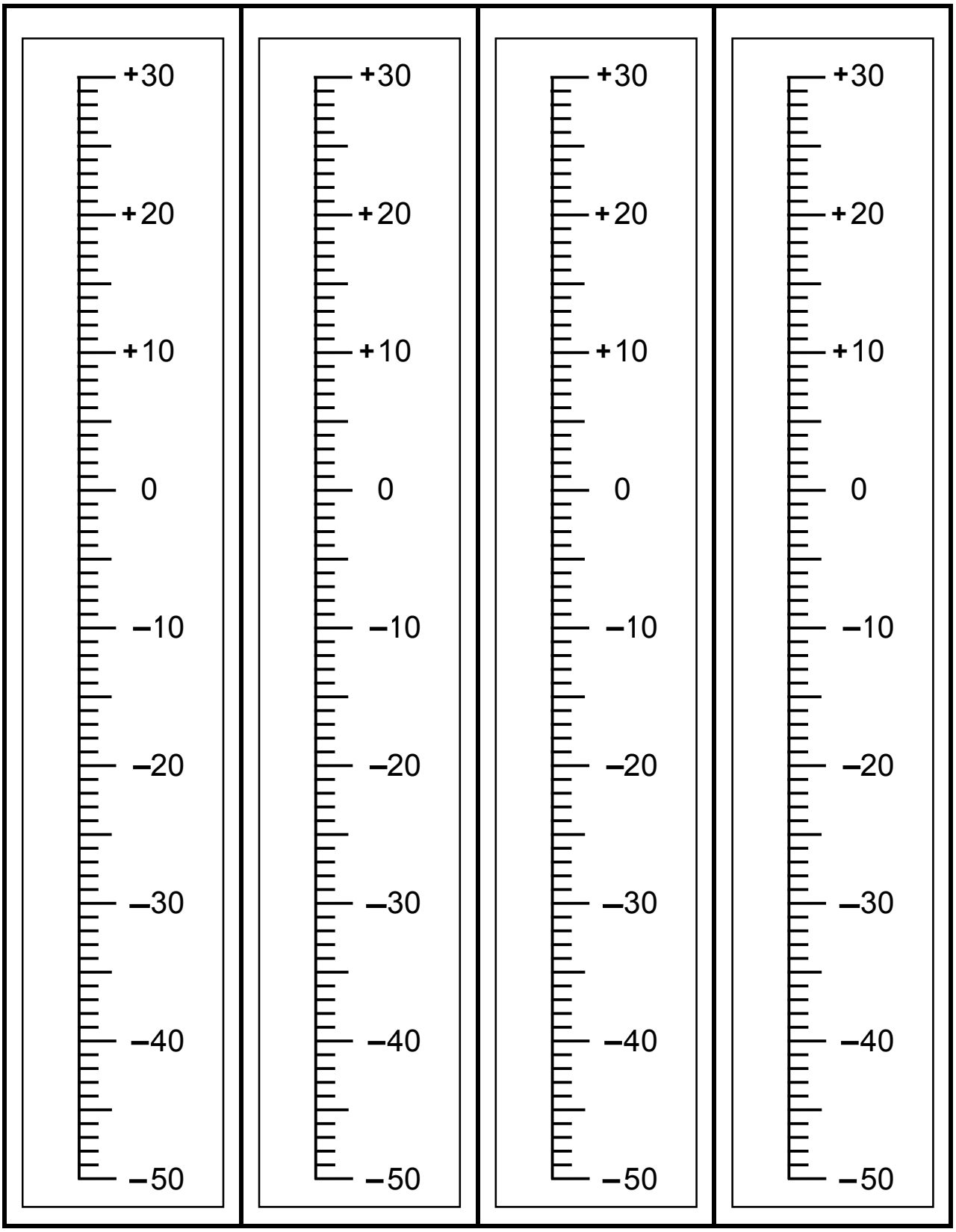
Temperature Scales



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Card Set: Introduction to Negative Numbers

London **+13°C**	**Paris** **+20°C**
Vancouver **−11°C**	**Beijing** **−5°C**

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Card Set: City Temperatures

Anchorage _ _ _ _ _ _ _ _	**New York** **–5°C**
Washington _ _ _ _ _ _ _ _	**Fairbanks** _ _ _ _ _ _ _ _
New Orleans **+10°C**	**Honolulu** **+25°C**
Las Vegas **+20°C**	**Detroit** _ _ _ _ _ _ _ _

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Card Set: Changes in Temperature

Washington to Las Vegas _ _ _ _ _ _ _ _ _ _ _	Washington to New Orleans **−10°C**
New Orleans to Las Vegas _ _ _ _ _ _ _ _ _ _ _	Anchorage to New York _ _ _ _ _ _ _ _ _ _ _
Honolulu to Anchorage **−35°C**	New York to Detroit **+15°C**
Detroit to Honolulu _ _ _ _ _ _ _ _ _ _ _	Las Vegas to Fairbanks **−45°C**
Anchorage to Washington **+30°C**	Fairbanks to Honolulu **+50°C**

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Card Set: Changes in Temperature (Continued)

– – – – – – – – – – – – – – – – – – $-5 - (+25)$ **= –30°C**	– – – – – – – – – – – – – – – – – – $-10 - (-25)$ **= +15°C**

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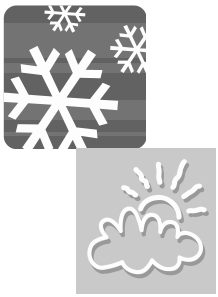
Temperature Changes (Again)

1.

The temperature was +5°C at midday.

By evening, the temperature was −3°C.



(a) Some of these calculations show how to figure out the temperature change.

Circle any that apply.

5 − 3	−3 − (+5)	− 5 − 3	(−5) + 3

(b) What was the change in temperature? Explain your answer.

2.

The temperature was −2°C at midnight.

By the next day, the temperature had risen by 9°C.

(a) Some of these calculations show how to figure out the temperature the next day.

Circle any that apply.

−2 − 9	9 + (−2)	(−9) − (−2)	(−2) + 9

(b) What was the temperature the next day? Explain your answer.

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3. Here is a calculation:

$$9 - (-4)$$

(a) Figure out the answer to the calculation:

(b) Which of these story questions fits the calculation?

Circle any that apply.

A	At midday, the temperature was 9°C. The temperature then fell by 4°C. What was the final temperature?
B	At midnight, the temperature was −4°C. By next morning, the temperature was 9°C. What was the change in temperature?
C	It was really cold at midnight. During the morning, the temperature rose by 9°C. By midday, the temperature was −4°C. What was the temperature at midnight?

Explain your answer:

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Temperature Changes

London
+13°C

Beijing
–5°C

Paris
+22°C

Vancouver
−11°C

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Instructions for Working Together

1. Take turns to place cards.
2. When it is your turn, connect two *City Temperature* cards using a *Changes in Temperature* arrow *card*.
   - Figure out any missing temperatures.
   - Explain your calculations.
3. If others in your group disagree, they should explain why. Then figure out the answer together.
4. When you have reached an agreement, write your solution in the box on the card.

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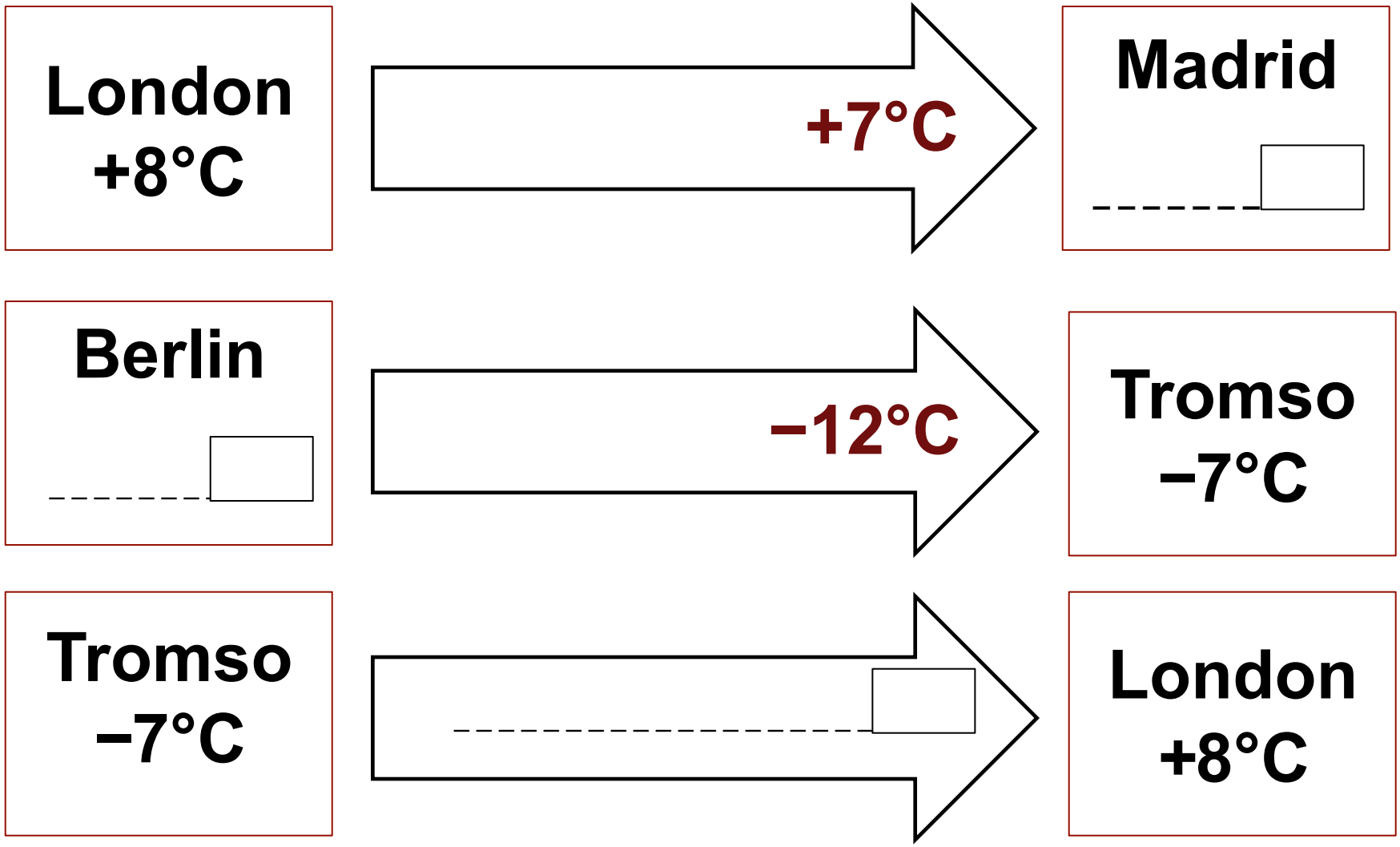
More Temperature Changes



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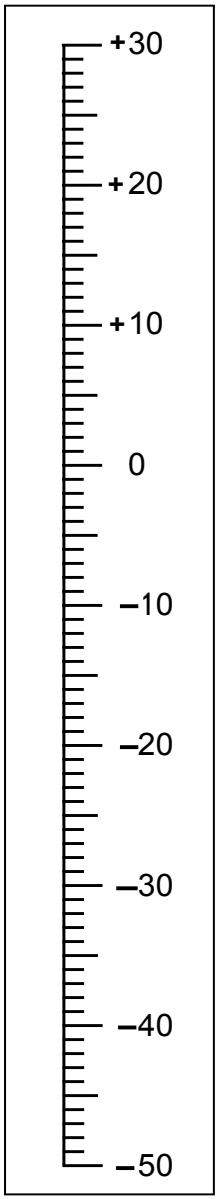
Temperature Scale