Mathematics Assessment Project

Formative Assessment Lesson Materials

Solving A Real-Life Problem: *Baseball Jerseys*

MARS Shell Center
University of Nottingham & UC Berkeley
Alpha Version

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Solving A Real-Life Problem: *Baseball Jerseys*

Mathematical goals

This lesson unit is intended to help you assess how well students are able to:

- Interpret a situation and represent the variables mathematically.
- Select appropriate mathematical methods to use.
- Explore the effects of systematically varying the constraints.
- Interpret and evaluate the data generated and identify the breakeven point, checking it for confirmation.
- Communicate their reasoning clearly.

Common Core State Standards

This lesson involves a range of *mathematical practices* from the standards, with emphasis on:

3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.

This lesson also asks students to select and apply mathematical content from across the grades, including the *content standards*:

7.EE Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

Introduction

This lesson is designed to enable students to develop strategies for solving real-life problems. .

- Before the lesson students attempt to solve the problem individually. You then review their work and formulate questions for students to answer in order to improve their solutions.
- At the start of the lesson, students work alone answering your questions.
- Students are then grouped and engage in a collaborative discussion of the same task.
- In the same small groups, students are given some sample solutions to evaluate and comment on.
- In a whole class discussion, students explain and compare the alternative solution strategies they have seen and used.
- Finally, students review what they have learnt.

Materials required

- For each individual student, you will need a calculator and a copy of the assessment task: *Baseball Jerseys,* and the questionnaire *How Did You Work?*
- Each small group of students will need a large sheet of paper, and copies of all the *Sample responses to discuss*.

Graph paper should be kept in reserve and used only when requested.

There are some projector resources to support class discussions.

Time needed

Approximately 15 minutes before the lesson, a one-hour lesson and a follow-up homework.

Timings given are only approximate. Exact timings will depend on the needs of the class.

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Before the lesson

Assessment task*: Baseball Jerseys* (15 minutes)

Set this task, in class or for homework, a few days before the formative assessment lesson. This will give you an opportunity to assess the work, and to find out the kinds of difficulties students have with it You will then be able to target your help more effectively in the follow-up lesson.

Give each student a copy of the assessment task *Baseball Jerseys*.

> *Read through the questions and try to answer them as carefully as you can. Show all your working, so that I can understand your reasoning.*
>
> *As well as trying to solve the problem, I want you to see if you can present your work in an organized and clear way.*

It is important that students are allowed to answer the questions without your assistance, as far as possible.

Students should not worry too much if they cannot understand or do everything, because in the next lesson they will engage in a similar task, which should help them. Explain to students that by the end of the next lesson, they should expect to answer questions such as these confidently. This is their goal.

Baseball Jerseys

Bill is going to order new jerseys for his baseball team.

The jerseys will have the team logo printed on them.

Bill asks two local companies to give him a price.

'Print it' will charge $21 for each jersey.

'Top Print' has a one-time set up fee of $45 and then charges $18 for each jersey

Bill decides to order 30 jerseys.

1. Use math to advise Bill which company to choose.

2. How many jerseys would Bill need to buy for the price from 'Top Print' to be less than the price from 'Print It'?

3. A third company states it charges $320 for a pack of jerseys.

   Figure out two possible ways of pricing the jerseys. For each of your prices, state how many jerseys are in the pack. Make sure one way includes a set up fee.

Students who sit together often produce similar answers, and then when they come to compare their work, they have little to discuss. For this reason, we suggest that when students do the task individually, you ask them to move to different seats. Then at the beginning of the formative assessment lesson, allow them to return to their usual seats. Experience has shown that this produces more profitable discussions.

Assessing students' responses

Collect students' responses to the task. Make some notes on what their work reveals about their current levels of understanding and their different problem solving approaches.

We suggest that you do not score students' work. The research shows that this will be counterproductive, as it will encourage students to compare their scores, and will distract their attention from what they can do to improve their mathematics.

Instead, help students to make further progress by summarizing their difficulties as a series of questions. Some suggestions for these are given on the next page. These have been drawn from common difficulties observed in trials of this unit.

We suggest that you write a selection of these questions on each piece of student work. If you do not have time, select a few questions that will be of help to the majority of students. These can be written on the board at the beginning of the lesson.

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Common issues:	Suggested questions and prompts:
Student has difficulty getting started	• *What do you know?* • *What do you need to find out?*
Student interpretation of the information is incomplete For example: The student may not have taken into account the setting up cost of $45 by 'Top Print'.	• *Write, in your own words all the information you are given.* • *What numbers in the task are fixed?* • *How can you check your answer?* • *What can you vary?*
Students work is unsystematic For example: The student writes 'Print It' 5 & 'Top Print' 10; 10 cost more etc.	• *Can you organize the costs of different numbers of jerseys made by the two companies in a systematic way?* • *What would be sensible values to try? Why?* • *Can you organize your work in a table?*
Students work is poorly presented For example: The student presents the work as a series of unexplained numbers and/or calculations. Or: The student draws a table without headings Or: The student circles or underlines numbers and it is left to the reader to work out why this is the answer as opposed to any other calculation.	• *Would someone unfamiliar with your type of solution easily understand your work?* • *Have you explained how you arrived at your answer?*
Student has technical difficulties when using graphs or equations For example: The student inaccurately plots lines, does not label the axis, or does not explain the purpose of the graph. Or: The student makes a mistake when solving the equation.	• *Would someone unfamiliar with your type of solution easily understand your work?* • *How can you check your answer?* • *How do your answers help you solve the problem?* • *Does your answer seem sensible?*
Student figures out an unrealistic values (Q.3) For example: The student writes the pack contains two jerseys, each costing $165. Or: The set-up fee is $230.	• *Are your prices realistic?* • *Is your set-up fee realistic?*
Student correctly answers all the questions Student needs an extension task.	• *Can you now use a different method? For example a table or graph or algebra?* • *Is this method better than your original one? Why?*

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Suggested lesson outline

Introduction: Baseball Jerseys (10 minutes)

Return the assessment task to the students.

If you have not added questions to individual pieces of work, then write your list of questions on the board. Students are to select questions appropriate to their own work, and spend a few minutes answering them.

> *Recall what we were looking at in a previous lesson. What was the task?*
>
> *I have read your solutions, and I have some questions I would like you to think about.*
>
> *At this point you do not need to do the math, but write a few sentences describing how you could improve your work.*

This is an opportunity for students to review their work.

Whilst they are doing this listen and watch students carefully. Are students able to reflect on their own work and suggest useful, detailed improvements? Ask questions that help students to clarify their thinking.

Collaborative work (20 minutes)

Organize the class into small groups of two or three students.

Give each group a fresh piece of paper.

Deciding on a Strategy

Ask students to share their ideas about the task, and plan a joint solution.

> *I want you to share your work with your group.*
>
> *Take it in turns to explain how you did the task and how you now think it could be improved.*
>
> *Listen carefully to any explanation. Ask questions if you don't understand or agree with the method. [You may want to use some of the questions I have written on the board.]*
>
> *I want you to plan a joint approach that is better than your separate solutions.*

Once students have evaluated the relative merits of each approach ask students to write their strategy on one side of their sheet of paper.

Slide 1 of the projector resource summarizes these instructions.

Implementing the Strategy

Students are now to write their joint solution on the second side of their sheet of paper.

While students work in small groups you have two tasks, to note different student approaches to the task and to support student problem solving.

You can then use this information to focus a whole class discussion towards the end of the lesson. Note any common mistakes. For example; are students consistently using all the given information? Which math do they choose to use? How do they use it? Attend in particular to the students' mathematical decisions. Do they track their progress in use of their chosen mathematics? Do they notice if they have chosen a strategy that does not seem to be productive? If so, what do they do?

Try not to make suggestions that move students towards a particular approach to this task. Instead, ask questions that help students to clarify their thinking, promote further progress and encourage students to develop self-regulation and error detection skills.

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You may find that some students find it difficult to keep more than one piece of information in mind at the same time. For example, you may ask them to consider these two questions:

If Bill were to order 10 jerseys from 'Print It', how much would they cost?

If Bill were to order 10 jerseys from 'Top Print', how much would they cost?

Students who organize their work into a table may like to use column headings for 'Number of jerseys', "Cost for Print It' and 'Cost for Top Print' which they can use.

How can you make your table clearer to someone who is unfamiliar with the math?

To further help students struggling with the task:

- Use the questions in the Common Issues table to support your own questioning or, if the whole class is struggling on the same issue, then write relevant questions on the board and hold a mini-discussion.
- Ask students who performed well on the assessment to help struggling students.
- Hand out two pieces of sample work.

Collaborative analysis of Sample responses to discuss (20 minutes)

After students have had sufficient time to solve the problem, give each group of students copies of all *Sample responses to discuss* and ask for written comments. This task gives students an opportunity to evaluate a variety of possible approaches to the task, without providing a complete solution strategy.

Imagine you are the teacher and have to assess students response to the second question.

Correct the work and write comments on the accuracy and organization of each response.

To encourage students to do more than check to see if the answer is correct, ask them to answer the questions below each sample piece of work.

Encourage students to focus on the math of the student work, not whether the student has neat writing etc.

There may not be time for all groups to look at all solutions, and it is not essential that they do. If so, be selective about what you hand out. For example, groups that have successfully completed the task using one method will benefit from looking at different approaches. Other groups that have struggled with a particular approach may benefit from seeing a student version of the same strategy.

During the small group work, support the students as before. Also, check to see which of the explanations students find more difficult to understand. Note similarities and differences between the sample approaches and those the students took in the small group work.

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Whole class discussion (10 minutes)

Now hold a whole class discussion to consider the different approaches used in the sample work. Focus the discussion on parts of the small group tasks students found difficult. Ask the students to compare the different solution methods.

Which approach did you like best? Why?

Which approach did you find most difficult to understand?

To critique the different strategies use the questions on the sheets *Sample responses to discuss* and Slides 2 - 5 of the projector resource.

Danny has found an effective way to organize his work; using a table. He has made some arithmetical mistakes in this table: ten jerseys at 'Top Print' should be $225, and 20 jerseys at 'Print It' should be $420. However, his reasoning is correct and these errors do not change his conclusions.

Why has Danny calculated the cost of 16 jerseys?

Number of Jerseys	Cost at 'Print it'	Cost at Top Print
5	$105	$135
10	$210	$245
15	$315	$315 ←
20	$430	$405
16	$336	$333

Top print cheaper more than 15 Jerseys.

Jeremiah has tried an algebraic approach, and he has found a correct solution for the break-even point. Equalities rather than inequalities have been used in his equation. However, he clearly understands the situation because he checks the costs for more than 15 jerseys.

What has Jeremiah figured out?

Should he be using the equal sign in his work?[No, he should use the inequality sign.]

Print it cost $21n
Top Print cost $45+18n
$n=30$
Print it cost $= 21 \cdot 30 = \$630$
Top Print cost $= 45 + 18 \cdot 30 = \$585$
$21n = 45 + 18n$
$3n = 45$
$n = 15$
$n = 16$ Print it $336 Top Print $333

Bella has tried a completely different approach.

She correctly shows that buying 30 jerseys from 'Print It' costs more than buying them from 'Top print'. She reasons that once the setting up cost of $45 has been paid, ($45 ÷ $3 = 15) 'Top Print' is $3 cheaper **per jersey**.

What does '$3 cheaper' mean?

30 at $21 = $630
30 at $18 + $45 = $585
Print it $45 more
He would have to buy 15 Jerseys because it would take 15 to get the $45 set up cost gone. Then Top Print $3 cheaper.

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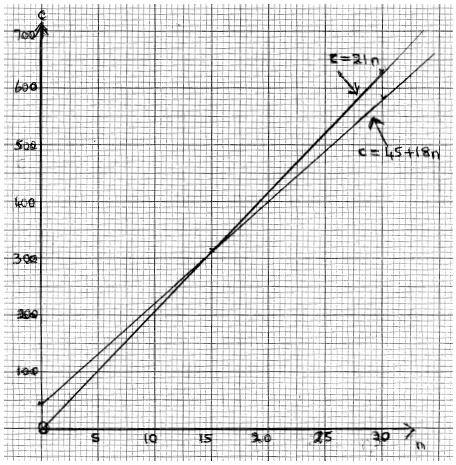
Tanya writes equations for the costs of buying jerseys from the two companies.

$c = 21n$ and $c = 45 + 18n$.

She draws two line graphs representing the two equations. She does not explain that the two graphs intersect at the point (15, 315), when the costs for the two companies are equal. The graph shows that for values of n less than 15, the cost for 'Print It' is less than 'Top Print'. For values of n greater than 15, the cost for 'Print It' is more than 'Top Print'.



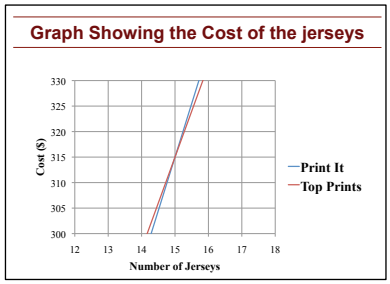
What does the intersection of the two lines mean?

How could you improve the accuracy of the answer? [Change the scale.]

Has anyone drawn a graph with a different scale?

You may want to show students the graph on Slide 6 of the projector resource.

The scales for each axis have been changed, however the accuracy is still an issue.



Does this new scale tell you the cost of jerseys?

In this case, what are the disadvantages, of using a graph?

Review solutions to *Baseball Jerseys* (10 minutes)

Give the questionnaire *How Did You Work?* to each student.

Ask students to and complete the review questionnaire.

Some teachers set this task as homework.

The questionnaire may help student monitor and review their progress during and at the end of an activity.

Solutions

1. Cost of 30 jerseys from: "Print It": 30 × 21 = \$630

   "Top Print": 45 + 30 × 18 = \$585.

2. For one or two jerseys, the one-off setting up cost means that the cost of each jersey is much more expensive from "Top Print". As the number of jerseys increases the difference in the costs becomes smaller. For 15 jerseys the costs are equal, and for more than 15 the cost at "Top Print" becomes less than at "Print It".

   Tabular method:

Number of jerseys	Cost at "Print It"	Cost at "Top Print"
1	\$21	\$63
2	\$42	\$81
5	\$105	\$135
10	\$210	\$405
15	\$315	\$315
20	\$420	\$405
25	\$525	\$495
16	\$336	\$333

   Algebraic method:

   "Top Print" jerseys cost lesson than "Print It" jerseys when:

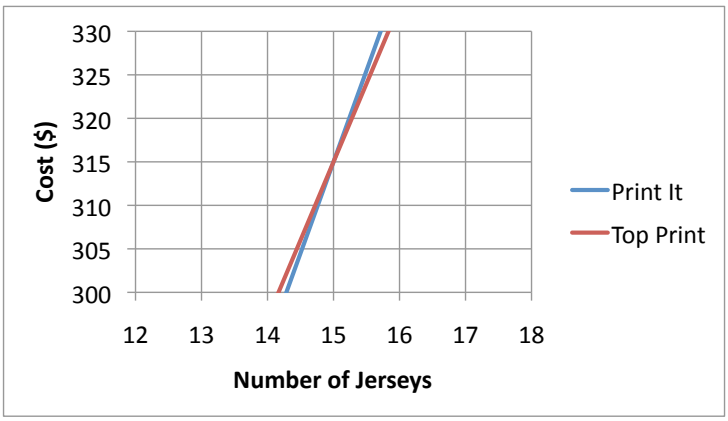
   $18n + 45 \leq 21n$ n is the number of jerseys.

   $3n \geq 45$

   $n \geq 15$

   Graphical method:

   (It is difficult to accurately read off values from this graph.)



3. There are many ways the student could answer this question. For example, each jersey costs \$15, 20 are bought and there is a \$20 set-up fee or each jersey costs \$20, 16 are bought and there is no set-up fee.

Baseball Jerseys

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Bill decides to order 30 jerseys.

1. Use math to advise Bill which company to choose.

2. How many jerseys would Bill need to buy for the price from ‘Top Print’ to be less than the price from ‘Print It’?

3. A third company states it charges $320 for a pack of jerseys.

   Figure out two possible ways of pricing the jerseys. For each of your prices, state how many jerseys are in the pack. Make sure one way includes a set up fee.

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Sample responses to discuss – Danny

Number of Jerseys	Cost at 'Print it'	Cost at Top Print
5	$105	$135
10	$210	$245
15	$315	$315 ←
20	$430	$405
16	$336	$333

Top print cheaper more than 15 Jerseys.

What Math did Danny do well?

Can you find two arithmetical errors in Danny's work?

Do these errors effect his conclusion?

In what ways might the work be improved?

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Sample responses to discuss - Jeremiah

Print it cost $21n

Top print cost $45+18n

$n=30$

Print it cost $= 21 \cdot 30 = \$630$

Top print cost $= 45 + 18 \cdot 30 = \$585$

$21n = 45 + 18n$

$3n = 45$

$n = 15$

$n = 16$ print it $336 Top print $333

What Math did Jeremiah do well?

Is Jeremiah's solution complete?

In what ways might the work be improved?

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Sample responses to discuss - Bella

30 at $21 = $630
30 at $18 + $45 = $585
Print it $45 more
He would have to buy 15 Jerseys because it would take 15 to get the $45 set up cost gone. Then TopPrint $3 cheaper.

What Math did Bella do well?

Is Bella's reasoning correct? Explain your answer

In what ways might the work be improved?

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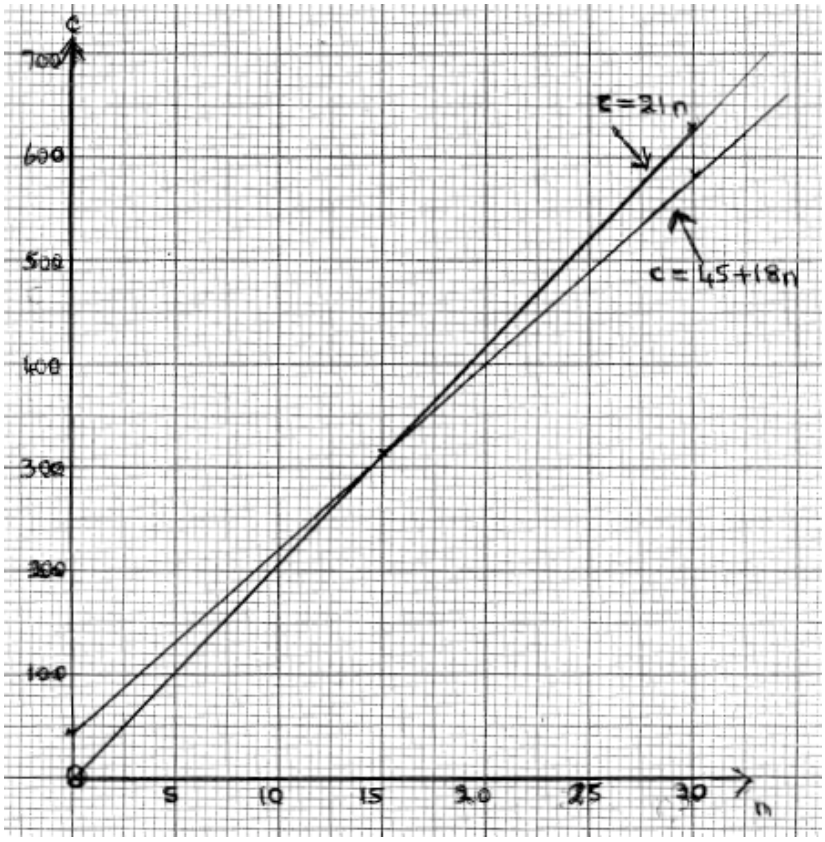
Sample responses to discuss – Tanya



What Math did Tanya do well?

What is not clear about the Tanya's answer?

In what ways might the work be improved?

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How Did You Work?

Tick the boxes and complete the sentences that apply to your work.

1. Our group work was better than my own work ☐

   Our joint solution was better because .....

   .....

2. We checked our solution ☐

   We checked our solution by .....

   .....

3. Our solution is similar to one of the sample responses ☐ **OR** Our solution is different from **all** of the sample responses ☐

   Our solution is similar to ..... *(add name of the student)*

   I prefer **our solution / the student's solution** (*circle)*

   This is because .....

   .....

   .....

   Our solution is different from all of the sample responses

   because .....

   .....

   .....

   .....

4. If you taught this lesson to another class, what advice would you give them about potential pitfalls?

   .....

   .....

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Planning a Joint Solution

1. Take it in turns to explain how you did the task and how you now think it could be improved.

2. Listen carefully to explanations.
   - Ask questions if you don't understand.
   - Discuss with your partners:
     - What you like/dislike about your partners' math.
     - Any assumptions your partner has made.
     - How their work could be improved.

3. Once everyone in the group has explained their solution, plan a joint approach that is better than each of the separate solutions.

4. Write an outline of your plan on one side of your sheet of paper

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Sample responses to discuss – Danny

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Top print cheaper more than 15 Jerseys.

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Sample responses to discuss - Bella

30 at \$21 = \$630

30 at \$18 + \$45 = \$585

Print it \$45 more

He would have to buy 15 Jerseys because it would take 15 to get the \$45 set up cost gone. Then TopPrint \$3 cheaper.

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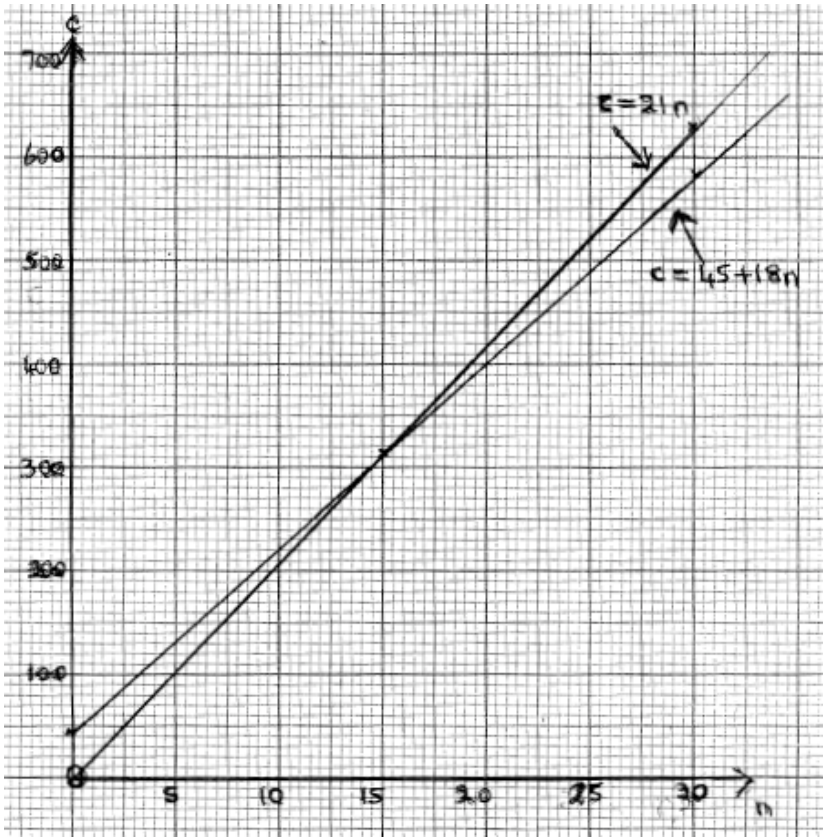
Sample responses to discuss – Tanya



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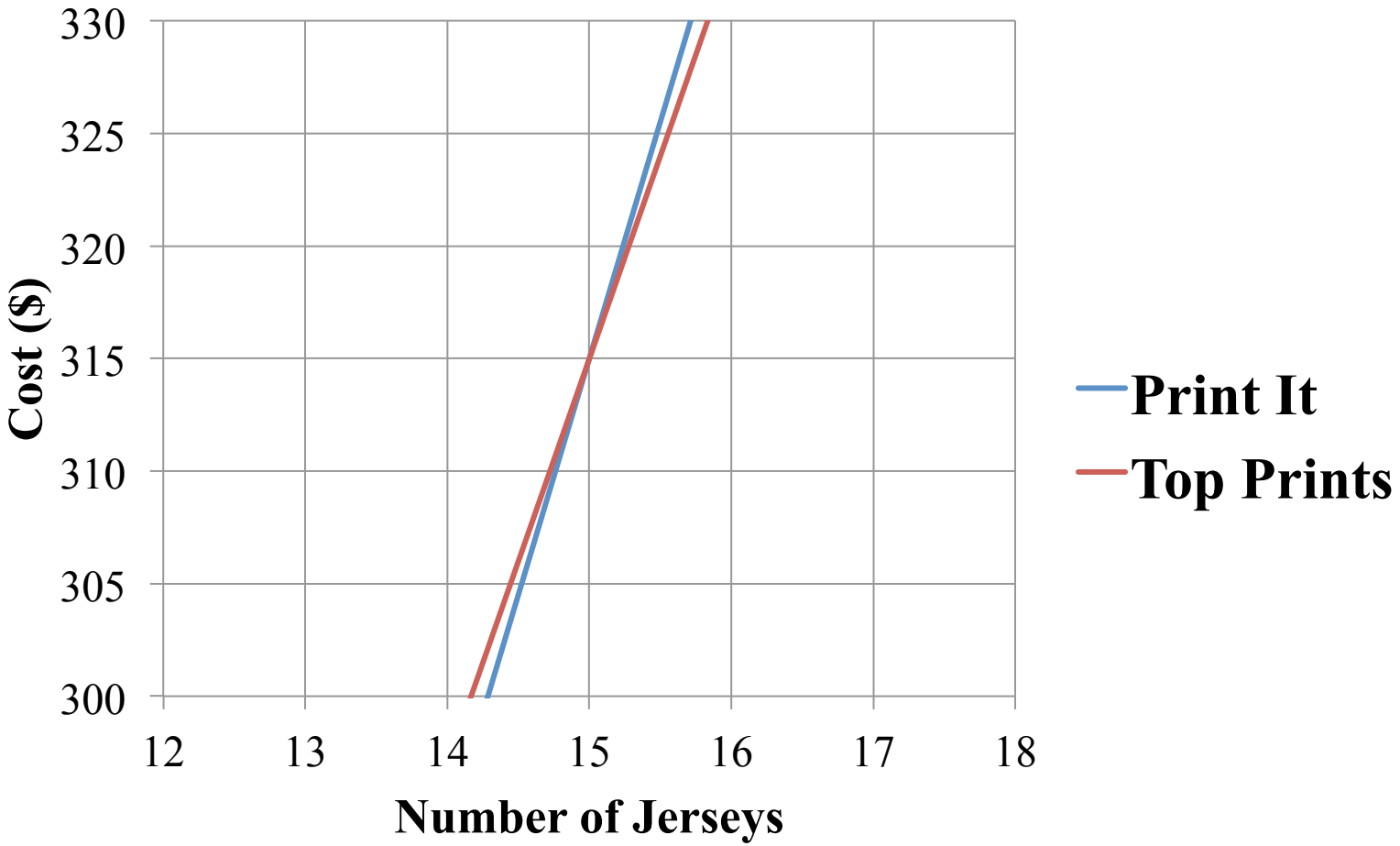
Graph Showing The Cost Of The Jerseys



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