Mathematics Assessment Project

Formative Assessment Lesson Materials

# Solving Linear Equations in One Variable

MARS Shell Center
University of Nottingham & UC Berkeley
Alpha Version

Please Note:

These materials are still at the "alpha" stage and are not expected to be perfect. The revision process concentrated on addressing any major issues that came to light during the first round of school trials of these early attempts to introduce this style of lesson to US classrooms. In many cases, there have been very substantial changes from the first drafts and new, untried, material has been added.

We suggest that you check with the Nottingham team before releasing any of this material outside of the core project team.

If you encounter errors or other issues in this version, please send details to the MAP team c/o map.feedback@mathshell.org.

© 2012 MARS University of Nottingham

# Solving Linear Equations in One Variable

## Mathematical goals

This lesson unit is intended to help you assess how well students are able to:

- Solve linear equations in one variable with rational number coefficients.
- Collect like terms.
- Expand expressions using the distributive property.
- Categorize linear equations in one variable as having one, none, or infinitely many solutions.

It also aims to encourage discussion on some common misconceptions about algebra.

## Common Core State Standards

This lesson involves *mathematical content* in the standards from across the grades, with emphasis on:

8.EE: Expressions and Equations.

This lesson involves a range of *mathematical practices*, with emphasis on:

1. Make sense of problems and persevere in solving them.
3. Construct viable arguments and critique the reasoning of others.

## Introduction

The lesson unit is structured in the following way:

- Before the lesson, students work individually on an assessment task that is designed to reveal their current understanding and difficulties. You then review their work, and create questions for students to answer in order to improve their solutions.
- After a whole-class introduction, students work in small groups on a collaborative discussion task, categorizing equations based on the number of solutions. Throughout their work, students justify and explain their thinking and reasoning.
- Finally, students return to their original task and try to improve their own individual responses.

## Materials required

- Each student will need two copies of the assessment task*, Too Good To Be True?* a mini-whiteboard, a pen, and, an eraser.
- Each small group of students will need *Card Set*: *Always, Sometimes, or Never True?* (cut into cards before the lesson), a marker, a glue stick, and a large sheet of paper for making a poster.
- There is a projector resource to support the whole-class introduction.

You may want to copy the *Card Set* onto transparencies to be used on an overhead projector to support whole-class discussions.

## Time needed

Fifteen minutes before the lesson for the assessment task, a sixty-five minute lesson, and ten minutes in a follow-up lesson (or for homework). Timings given are only approximate. Exact timings will depend on the needs of the class.

© 2012 MARS University of Nottingham 

# Before the lesson

## Assessment task: *Too Good To Be True?* (15 minutes)

Have the students complete this task, in class or for homework, a day or more before the formative assessment lesson. This will give you an opportunity to assess the work, and to find out the kinds of difficulties students have with it. You will then be able to target your help more effectively in the follow-up lesson.

Give each student a copy of the assessment task:

> *Spend fifteen minutes working individually, answering these questions. Make sure you explain your answers really clearly.*

It is important that students are allowed to answer the questions without assistance, as far as possible.

Students should not worry too much if they cannot understand or do everything, because in the next lesson they will work on a similar task, which should help them. Explain to students that by the end of the next lesson, they should be able to answer questions such as these confidently. This is their goal.

**Too Good To Be True?**

1. Amy and Ben are trying to answer this question:

When is the equation $5 - x = 6$ true?

They decide to compare their work.

Amy

$5 - x = 6$
so $x = 6 - 5 = 1$
so it is true when $x = 1$

Ben

You are taking a number away from 5 so the answer can't be bigger Therefore it is never true.

Both Amy and Ben have made a mistake. Describe where they have gone wrong:

Amy: .....

Ben: .....

Show how you would work it out:

.....

## Assessing students' responses

Collect students' responses to the task, and make some notes on what their work reveals about their current levels of understanding. The purpose of doing this is to forewarn you of the difficulties students will experience during the lesson itself, so that you may prepare carefully.

We suggest that you do not write scores on students' work. The research shows that this is counterproductive, as it encourages students to compare scores, and distracts their attention from what they can do to improve their mathematics.

Instead, help students to make further progress by asking questions that focus their attention on aspects of their work. Some suggestions for these are given on the next page. These have been drawn from common difficulties observed in trials of this unit.

We suggest that you write your own list of questions, based on your own students' work, using the ideas that follow in the *Common issues* table. We strongly recommend that you write questions on each student's work. If you do not have time to do this, write a few questions that will help the majority of students. These can then be displayed on the board at the end of the lesson.

2. Amy and Ben try a different equation.

$$8x - 6 = 2x$$

Look at their work and comment on the approach used, identifying any mistakes they have made.

Amy's work:

8x and 6 are not "like terms"
If the equation was $8x - 6x = 2x$
then it would be always true

Ben's work:

Substituting $x=0$ gives $0-6 \neq 0$
" $x=1$ " $8-6=2$ ✓
It doesn't work for all values of $x$
So it is false

Show how you would work it out:

.....

| **Common issues:** | **Suggested questions and prompts:** |
|---|---|
| **Student fails to explain why Amy and Ben are incorrect (Q1)** | • *How can you check that Amy's value of x satisfies the equation?*<br>• *Consider this: three subtract two equals one, three subtract one equals two, three subtract zero equals three. Can you see any patterns? What comes next? Three subtract something equals four?* |
| **Student understands both Amy and Ben are incorrect, but does not provide a correct solution** | • *What math can you use to work out if there is a value for x that satisfies the equation? [Guess and check or make x the subject of the equation.]* |
| **Student incorrectly uses the equal sign**<br>For example: The student writes $5 - x = 6 = x = -1$. | • *Carefully check your work. In your head say the math you've written. Does it make sense if you read it from left to right?* |
| **Student solves the equation to give $x = -1$ correctly and states the equation is true** | • *Is this equation always true?*<br>• *How many solutions does this equation have?*<br>• *How do you know?* |
| **Student assumes Amy is correct (Q2)** | • *Give x a value. Are the terms alike now? Can you now subtract 6 from 8x?*<br>• *Can you find a value for x that makes the equation true?* |
| **Students assumes Ben is correct (Q2)** | • *How many solutions does an equation need to have, to be true?*<br>• *Is x = 1 the only solution to this equation? How do you know?* |
| **Student completes the task** | • *Can you think of a different way of showing when the equation is true? Which method is the most convincing? Why do you think this is?*<br>• *Can you write down a new equation, that is always/never true? How could you prove this?*<br>• *Can you think of an equation that is sometimes true, but has more than one solution?* |

© 2012 MARS University of Nottingham 

## Suggested lesson outline

### Whole-class introduction (15 minutes)

Give each student a mini-whiteboard, a pen, and an eraser. Maximize participation in the discussion by asking all students to show you their solutions on their mini-whiteboards.

This introduction will provide students with a model of how they should justify their solutions in the collaborative activity.

Display slide 1 of the projector resource:

**True or False?**

$$4x + 1 = 3$$

Can you give me a value for $x$ that makes this equation **false**?

Show the calculations that explain your answer.

Students shouldn't have any problems with finding a suitable value for $x$, but may not be too adventurous in their choices. Spend some time discussing all the values given and the reasons for each choice, identifying any common choices, as well as any calculation errors.

Display slide 2 of the projector resource:

**True or False?**

$$4x + 1 = 3$$

Can you give me a value for $x$ that makes this equation **true**?

Show the calculations that explain your answer.

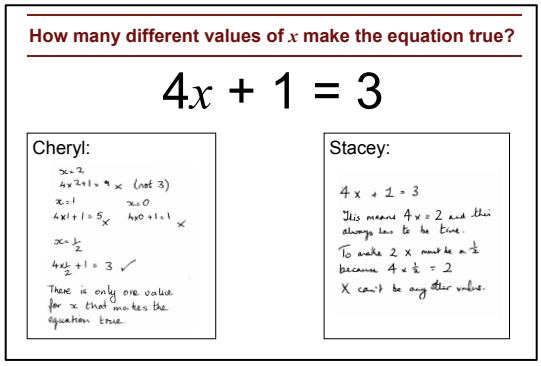
Students may struggle with this at first, especially if their chosen method is substituting values for $x$. Encourage students to explore fractions, decimals and negative numbers as well as positive whole numbers.

If students find a value for $x$, challenge them to consider if there are any other values of $x$. They should be encouraged to justify why this is the only value for $x$ that makes the equation true and how they can be sure of this.

> *Would we describe this equation as always true, never true or sometimes true? [Sometimes true.]*
>
> *When is it true? [When $x = ½$.]*
>
> *Are there any other values for $x$ that make the equation true? How do you know?*

© 2012 MARS University of Nottingham 

If students struggle to justify why there is only one value for $x$ that makes the equation true then show slide 3 of the projector resource:




Ask students to vote on which explanation they prefer. Ask one or two students to justify their preference.

Cheryl has used the method of guess and check. Once she has found a value for $x$ that works, she has assumed that this is the only value and so has not tried to find any others. She has not explained why this is the case.

Stacey has found the value of $x$ as ½ by "undoing" the equation. She has stated that $x$ can't be any other value, using her knowledge that the only number to give an answer of 2 when multiplied by 4 is ½. She has not used a formal algebraic proof.

Explain to students that during this lesson they will be considering a number of equations in the same way. They will be working in small groups to produce a poster, which shows each equation classified according to whether it is always, sometimes or never true.

> *What does always true mean? [The equation is true for any value of x.]*
>
> *What does never true mean? [There are no values of x that make the equation true.]*
>
> *For an equation that is sometimes true, how many values of x do you need? [Two. One value that makes the equation true, and one value that makes the equation false.]*

All of the equations are linear, so there will only ever be one solution for the equations that are sometimes true. Students may not be aware of this and should be encouraged to explore the number of solutions for different types of equation as appropriate.

### Collaborative activity: *Always, Sometimes, or Never True?* (30 minutes)

Ask students to work in groups of two or three.

Give each group *Card Set*: *Always, Sometimes, or Never True?*, a large sheet of paper, a marker, and a glue stick.

Ask students to divide their large sheet into three columns and head respective columns with the words: *Always True*, *Sometimes True*, *Never True.*

You may want to use slide 4 of the projector resource to display the following instructions.

> *You are now going to consider whether the equations on your desk are always true, sometimes true, or never true.*
>
> *In your groups, take turns to place a card in a column and justify your answer to your partner.*
>
> *If you think the equation is sometimes true, you will need to find values of x for which it is true.*
>
> *If you think the equation is always true or never true, you will need to explain how we can be sure that this is the case. Remember, showing it is true or never true for just a few values is not sufficient.*
>
> *Another member of the group should then either explain their reasoning again in his or her own words, or challenge the reasons you gave.*

*When the entire group agrees, glue the card onto your poster. Write the reason for your choice of category next to the card.*

The purpose of this structured work is to encourage each student to engage with their partner's explanations, and to take responsibility for their partner's understanding. Students should use their mini-whiteboards for calculations and to explain their thinking to each other.

It does not matter if students do not manage to place all of the cards. It is more important that everyone in each group understands the categorization of each card.

While students work in small groups you have two tasks: to make a note of student approaches to the task, and to support student reasoning.

**Make a note of student approaches to the task**

Listen and watch students carefully. In particular, listen to see whether students are addressing the difficulties they experienced in the assessment task. Are students struggling with fractions? Do students justify their categorization by using the commutative or distributive properties? Are students using guess and check or do they make $x$ the subject of the equation? Do students have a systematic strategy? Do students have problems with unlike terms? Do students misuse the equal sign? For example, students may write $3x - 5 = 2x = x = 5$.

You can use this information to focus the whole-class discussion towards the end of the lesson.

**Support student reasoning**

Encourage students to explain their reasoning carefully.

*You have shown the statement is true for this specific value of $x$. Now convince me it is always true for every number!*

*Can you use algebra to justify your decision for this card?*

*You say this equation is never true. How do you know for sure?*

You may want to use the questions in the *Common issues* table to help address misconceptions.

Encourage students to write an explanation for the placement of each card.

If one student has placed a card, challenge another student in the group to provide an explanation.

*Drew placed this card. Ellie, why does Drew think the card goes here?*

If the student is unable to answer this question, ask the group to discuss the work further. Explain that you will return in a few minutes to ask a similar question.

If the whole class is struggling on the same issue, you could write a couple of questions on the board and hold a whole class discussion.

If students finish the task quickly, ask them to create new examples and add them to the table.

## Whole-class discussion (20 minutes)

Ask students to display their work at the front of the room for all to see.

Organize a whole-class discussion about different methods of justification used for three or four equations, preferably at least one from each column. You may want to first select a card that most groups placed correctly as this may encourage good explanations. Then select one or two cards that most groups found difficult. Once one group has justified their choice for a particular equation, ask other students to contribute ideas of alternative approaches, and their views on which reasoning method was easier to follow.

*Why did you put this equation in this column? How else can you explain that decision?*

*Can anyone improve this explanation?*

*Which explanation do you prefer? Why?*

© 2012 MARS University of Nottingham 

Draw out issues you have noticed as students worked on the activity. Make specific reference to the misconceptions you noticed during the collaborative activity. You may want to use the questions in the *Common issues* table to support the discussion.

At this stage, you could check that the students know some key terms such as 'solution' and 'identity':

> *Can you tell me another name for an equation that is always true? [Identity.]*
> *What do we call the value of x for which an equation is true? [The solution.]*

### Follow-up lesson: improving individual solutions to the assessment task (10 minutes)

Return to students their original assessment, as well as a second blank copy of the task.

> *Look at your original responses, and think about what you have learned this lesson.*
>
> *Using what you have learned, try to improve your work.*

If you have not added questions to individual pieces of work, write your list of questions on the board. Students should select from this list only the questions they think are appropriate to their own work.

If you think at this stage of the lesson your students may lack the concentration needed for the assessment task, or you find you are running out of time, then you could set this task in the next lesson or for homework.

© 2012 MARS University of Nottingham 

## Solutions

**Assessment task**

1. $5 - x = 6$

   The equation $5 - x = 6$ is only true when $x = -1$.

   Amy has solved the equation $5 + x = 6$ instead of $5 - x = 6$.
   The second line should read: $-x = 6 - 5$ so $x = -1$.

   Ben has not considered the effects of subtracting a negative number.
   He has not tried to substitute values for $x$ or shown any algebraic manipulation.

2. $8x - 6 = 2x$

   The equation $8x - 6 = 2x$ is only true when $x = 1$.

   Amy has made a valid point about like terms, but has not looked beyond this to see if there is a value of $x$ that makes this equation true.

   Ben has found a solution for the equation using substitution.
   He has incorrectly assumed that the equation is false because it is not true for all values of $x$.

**Collaborative Activity:**

| Always true | Sometimes true | Never true |
|---|---|---|
| $3 + x = x + 3$ | $2 - x = x - 2$<br>True when $x = 2$. | $x + 5 = x - 3$ |
| $7x + 14 = 7(x + 2)$ | $6x = x$<br>True when $x = 0$. | $5x - 5 = 5(x + 1)$ |
| $\frac{2x + 4}{2} = x + 2$ | $4x = 4$<br>True when $x = 1$. | $2(x + 1) = 2x + 1$ |
| | $\frac{10}{2x} = 5$<br>True when $x = 1$. | |
| | $\frac{x}{2} = x$<br>True when $x = 0$. | |
| | $3x - 5 = 2x$<br>True when $x = 5$. | |

© 2012 MARS University of Nottingham 

# Too Good To Be True?

1. Amy and Ben are trying to answer this question:

When is the equation $5 - x = 6$ true?

They decide to compare their work.

Amy

$5 - x = 6$

so $x = 6 - 5 = 1$

so it is true when $x = 1$

Ben

You are taking a number away from 5 so the answer can't be bigger Therefore it is never true.

Both Amy and Ben have made a mistake. Describe where they have gone wrong:

Amy: .....

.....

.....

Ben: .....

.....

.....

Show how you would work it out:

.....

.....

.....

© 2012 MARS University of Nottingham 

2. Amy and Ben try a different equation.

$$8x - 6 = 2x$$

Look at their work and comment on the approach used, identifying any mistakes they have made.

Amy's work:

8x and 6 are not "like terms"
If the equation was $8x - 6x = 2x$
then it would be always true

Ben's work:

Substituting $x = 0$ gives $0 - 6 \neq 0$
" $x = 1$ " $8 - 6 = 2$ ✓
It doesn't work for all values of $x$
So it is false

Show how you would work it out:

© 2012 MARS University of Nottingham

# Card Set: Always, Sometimes, or Never True?

| | |
|---|---|
| 1 $2 - x = x - 2$ | 2 $3 + x = x + 3$ |
| 3 $x + 5 = x - 3$ | 4 $3x - 5 = 2x$ |
| 5 $\frac{x}{2} = x$ | 6 $2(x+1) = 2x + 1$ |
| 7 $6x = x$ | 8 $7x + 14 = 7(x+2)$ |
| 9 $\frac{10}{2x} = 5$ | 10 $\frac{2x+4}{2} = x + 2$ |
| 11 $5x - 5 = 5(x+1)$ | 12 $4x = 4$ |

© 2012 MARS University of Nottingham

# True or False?

$$4x + 1 = 3$$

Can you give me a value for $x$ that makes this equation **false**?

Show the calculations that explain your answer.


© 2012 MARS, University of Nottingham

# True or False?

$$4x + 1 = 3$$

Can you give me a value for $x$ that makes this equation **true**?

Show the calculations that explain your answer.


© 2012 MARS, University of Nottingham

# How many different values of $x$ make the equation true?

$$4x + 1 = 3$$

Cheryl:

$x = 2$

$4 \times 2 + 1 = 9$ ✗ (not 3)

$x = 1$ $\quad$ $x = 0$

$4 \times 1 + 1 = 5$ ✗ $\quad$ $4 \times 0 + 1 = 1$ ✗

$x = \frac{1}{2}$

$4 \times \frac{1}{2} + 1 = 3$ ✓

There is only one value for $x$ that makes the equation true.

Stacey:

$4x + 1 = 3$

This means $4x = 2$ and this always has to be true.

To make 2 x must be a $\frac{1}{2}$ because $4 \times \frac{1}{2} = 2$

X can't be any other value.


© 2012 MARS, University of Nottingham

# Always, Sometimes or Never True?

(1) In your groups, take turns to place a card in a column and justify your answer to your partner.

(2) If you think the equation is sometimes true, you will need to find values of $x$ for which it is true.

(3) If you think the equation is always true or never true, you will need to explain how we can be sure that this is the case.

(4) Another member of the group should then either explain that reasoning again in his or her own words, or challenge the reasons you gave.

(5) When the entire group agrees, glue the card onto your poster. Write the reason for your choice of category next to the card.


© 2012 MARS, University of Nottingham