Mathematics Assessment Project

Formative Assessment Lesson Materials

Using Dimensions:
Designing a Sports Bag

MARS Shell Center
University of Nottingham & UC Berkeley
Alpha Version

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Using Dimensions: *Designing a Sports Bag*

Mathematical goals

This lesson unit is intended to help you assess how well students are able to:

- Recognize and use common 2D representations of 3D objects.
- Identify and use the appropriate formula for finding the circumference of a circle.

Common Core State Standards

This lesson involves a range of *mathematical practices* from the standards, with emphasis on:

1. Make sense of problems and persevere in solving them.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
6. Attend to precision.

This lesson asks students to select and apply mathematical content from across the grades, including the *content standards*:

7.G: Draw, construct, and describe geometrical figures and describe the relationships between them.
Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.

Introduction

The unit is structured in the following way:

- Before the lesson, students attempt the *Designing a Sports Bag* task individually. You review their work, and write questions to help students improve their solutions.
- At the start of the lesson, students work individually, answering your questions.
- Next, students work collaboratively in small groups. Their task is to produce a better solution to the task together than they did individually. Then, working in the same small groups, students analyze some solutions written by students in another class.
- In a whole-class discussion, students compare and evaluate the sample solutions.
- To complete their work on the task, students spend ten minutes reviewing their individual solutions and writing about what they have learned.

Materials required

- Each individual student will need a copy of the task sheet, *Designing a Sports Bag*, some plain paper, a mini-whiteboard, pen and wipe.
- Each small group of students will need a new copy of the task sheet, *Designing a Sports Bag*, a sheet of poster paper, a marker, copies of the *Sample Responses to Discuss,* and a sheet of lined paper.
- Provide calculators, rulers, and squared paper for students who choose to use them.
- There are some projector resources to support whole-class discussion.

Time needed

Approximately twenty minutes before the lesson, a one-hour lesson and ten minutes in a follow-up lesson (or for homework.) Timings given are only approximate. Exact timings will depend on the needs of your class.

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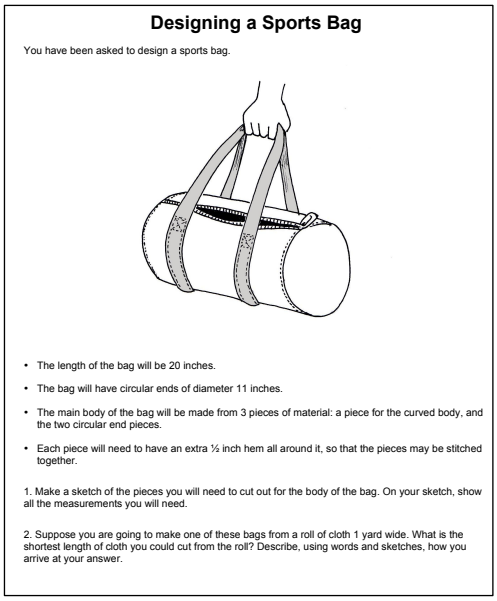
Before the lesson

Assessment task: *Designing a Sports Bag* (20 minutes)

Have the students do this task, in class or for homework, a day or more before the lesson. This will give you an opportunity to assess their work, and to find out the kinds of difficulties students have with it. You will then be able to target your help more effectively in the follow-up lesson.

Give each student a copy of the task, *Designing a Sports Bag,* and some plain paper.



Designing a Sports Bag

You have been asked to design a sports bag.

- The length of the bag will be 20 inches.
- The bag will have circular ends of diameter 11 inches.
- The main body of the bag will be made from 3 pieces of material: a piece for the curved body, and the two circular end pieces.
- Each piece will need to have an extra ½ inch hem all around it, so that the pieces may be stitched together.

1. Make a sketch of the pieces you will need to cut out for the body of the bag. On your sketch, show all the measurements you will need.

2. Suppose you are going to make one of these bags from a roll of cloth 1 yard wide. What is the shortest length of cloth you could cut from the roll? Describe, using words and sketches, how you arrive at your answer.

Introduce the task. Take time to help students to understand the problem context, which may be unfamiliar to some of them. There are projectable resources to help you do this.

> *Have you ever seen someone making a piece of clothing or a bag from fabric?*
>
> *Suppose you were making this bag. Ignore the handles. What pieces of fabric would you need to make the body of the bag?*
>
> *Pieces of fabric are cut from a long roll of cloth called a bolt. [Slide 1: Unrolling a Bolt of Fabric.]*
>
> *To make a bag, you take paper pattern pieces. You put the paper pattern pieces are pinned to the fabric, then you cut round them. After that, the fabric pieces can be sewn together. [Slide 2: Placing Paper Pattern Pieces, and Slide 3: Pinning and Cutting Before Sewing.]*

Focus students' attention on how different arrangements of pattern pieces use more or less fabric.

> *The bolt of fabric has a fixed width, in this case, a yard.*
>
> *You unroll the bolt and buy the length of fabric you need. You can choose how many inches to buy.*
>
> *How could you arrange these pattern pieces to use less fabric? [Slide 4: Arrange the Pattern Pieces on the Cloth; Slide 5: Minimize the Length of Fabric Purchased.]*

Introduce the idea of a hem.

> *How are all the pieces attached together?*
>
> *Does anyone know what to call the extra fabric around the edge of each piece of fabric? [Seam allowance or hem.]*
>
> *What happens to the hems once the fabric is sewn together? [They are hidden inside the garment.]*

Now explain what you are asking students to do. Make it clear to students that the images they see show a tote bag, which is not the one on which they'll work.

> *In your task, you be working on the pattern pieces for a sports bag. The bag will be cut from fabric and sewn together. You will need to think about the shapes you'd need, and which seam allowances to include on the paper pattern pieces.*
>
> *I would like you to spend fifteen minutes on your own, answering these questions as carefully as you can. Show all your work so that I can understand your reasoning.*
>
> *There will be a lesson [tomorrow] that will help you improve your work.*

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It is important that students are allowed to answer the questions without assistance, as far as possible. If students are struggling to get started, then ask questions that help them understand what is required, and the context of the questions, but make sure you do not do the task for them.

Students who sit together often produce similar answers, and then when they come to compare their work, they have little to discuss. For this reason we suggest that, when students do the task individually, you ask them to move to different seats. Then at the beginning of the formative assessment lesson, allow them to return to their usual seats. Experience has shown that this produces more profitable discussions.

When all students have made a reasonable attempt at the task, tell them that they will have time to revisit and revise their solutions later.

Assessing students' responses

Collect students' responses to the task. Make some notes on what their work reveals about their current levels of understanding, and their problem solving strategies.

We suggest that you do not score students' work. The research shows that this will be counterproductive, as it will encourage students to compare scores, and distract their attention from what they can do to improve their mathematics.

Instead, help students to make further progress by summarizing their difficulties as a list of questions. Some suggestions for these are given the *Common Issues* table below. We suggest that you make a list of your own questions, based on your students' work, using the ideas on the following page. You may choose to write questions on each student's work. If you do not have time to do this, select a few questions that will be of help to the majority of students. These can be written on the board at the beginning of the lesson.

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Common issues:	Suggested questions and prompts:
Student draws pattern pieces incorrectly For example: The student has not recognized that the body of the bag is formed from a rectangle.	• *Imagine you remove the ends of the bag, and unfold the center. If you lay that piece flat, what shape will it be?*
Student ignores assumptions and constraints in making the bag For example: The student omits a seam allowance on some edges, such as the length or width of the rectangle. Or: The student adds a seam allowance or seam allowances before calculating the length of the rectangle.	• *Have you used all the information in the question?* • *Where will the sides of the rectangle be attached? How?* • *Which of these circles is sewn to the length of the rectangle: the inner circle, or the outer? How does your answer affect the length of the rectangle you need to cut?*
Student does not show any calculations	• *How did you calculate the length of the rectangular piece?* • *Can you explain how you have calculated the outside measurements of the rectangle, including the seams?*
Student does not work to appropriate degree of accuracy For example: The student works to 4 decimal places, in a context that involves cutting cloth by hand. Or: The student rounds π to 3, reducing the length of the rectangle significantly.	• *How accurate do your answers need to be for the pieces to fit?* • *Do you think it is better to round up or round down when deciding the lengths to cut? Why?*
Student does not justify his/her answers adequately For example: The student does not provide a sketch showing hems. Or: The student does not show how the shortest length of cloth needed has been obtained. Or: The student's sketches do not include measurements.	• *Explain in writing how you would cut these pieces from a roll of cloth one yard wide, wasting the least material.* • *Show me clearly how the different shapes fit on the roll of cloth. Convince me that they fit, including the seams!*
Student completes the task	• *Explain how you can be sure that you have found the best possible arrangement, using least cloth. Would your solution change if the cloth were 60 inches wide? How?*

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99

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Suggested lesson outline

Introduction *Designing a Sports Bag* task (10 minutes)

Give each student a mini-whiteboard, pen and wipe. Begin the lesson by briefly reintroducing the problem:

> *Recall what we were working on previously. What was the task?*
>
> *I have had a look at your work, and I have some questions I would like you to think about.*
>
> *Read the questions, and write your answers on your mini-whiteboards.*

Return your students' work on *Designing a Sports Bag*. If you have not added questions to students' work, write a short list of your most common questions on the board. Students can then select a few questions appropriate to their own work and begin answering them.

Collaborative activity: Improving the solutions (20 minutes)

When students have made a reasonable attempt at the task on their own, organize them into groups of two or three. Give each group a piece of poster paper, and a marker pen. Ask students to have another go at the task, but this time, ask them to combine their ideas and make a poster to show their solutions.

> *I want you to share your work with a partner. First, take turns to explain how you approached the task, and how you think you could now improve your solution.*
>
> *Then produce a joint solution that is better than your individual responses.*

While students work in small groups you have two tasks: to note different approaches to the task, and to support their reasoning.

Note different student approaches to the task

Listen and watch students carefully and note different student approaches to the task. Do students choose appropriate dimensions when calculating missing measures? Do they consider all constraints in making the bag? Have they described their method and solution effectively? Are they working to an appropriate degree of accuracy? This information will help you focus for discussion at the end of the lesson.

Support student problem solving

Try not to make suggestions that move students towards a particular solution. Instead, ask questions that help students to clarify their thinking. The suggestions in the *Common Issues* table (above) may be helpful. If one group is struggling with an issue, you could refer them to a group who are more successful on that issue for help. If the whole class is struggling on the same issue, you could write a relevant question on the board, or hold a brief whole-class discussion.

Collaborative analysis of *Sample Responses to Discuss* (15 minutes)

When students have had sufficient time to attempt the problem in their groups, give each group copies of the *Sample Responses to Discuss* and a sheet of lined paper on which to work. This task gives students an opportunity to evaluate a variety of possible approaches to the task, without providing a complete solution strategy.

> *Here are some different solutions to the problem. I want you to work together on one solution at a time.*
>
> *Imagine you are the teacher.*
>
> - *What do you like and dislike about the work?*
> - *What is unclear about the work?*
> - *How could this student improve his or her solution?*

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There may not be time for all groups to look at all four *Sample Responses*, and it is not essential that they do so. It is more important that students provide a detailed evaluation of the work they are critiquing, and suggest ways in which it could be improved.

Plenary whole-class discussion: comparing different solution methods (15 minutes)

Organize a whole-class discussion comparing the *Sample Responses*. Collect comments and ask for explanations.

We are going to look at and compare the four solutions.

What is missing from Ben's diagrams?

Should hems be included when calculating the length of the rectangle? Why/why not?

How could Aisha improve her response?

Finally, ask students to evaluate and compare responses.

Which response was easiest to follow? Why?

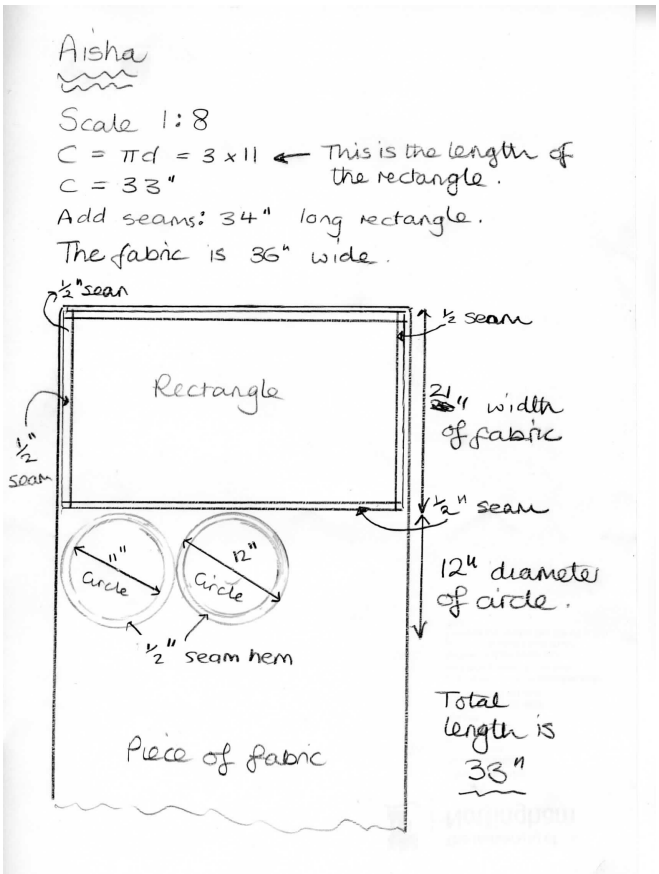
Which student's rounding methods are most appropriate for this context?

How accurately can you cut fabric?

Does any student justify his or her response to Q2 adequately?

What would a good justification for Q2 look like?

Some issues that might be discussed are given below:



Aisha has made accurate scale drawings of the main pattern pieces, labeled lengths, and shown the scale for Q2, but has not made separate drawings for Q1. She has added appropriate seam allowances.

Aisha explains her calculations, and why she is making them. She has labeled the diameter as 11", showing that she knows what the term means.

Aisha does not work to an appropriate degree of accuracy for the context. She rounds π to 3, significantly reducing the length of the rectangle that would be cut. The piece would be too short to sew around the circumference of an 11" circle, even including a ½" hem each end.

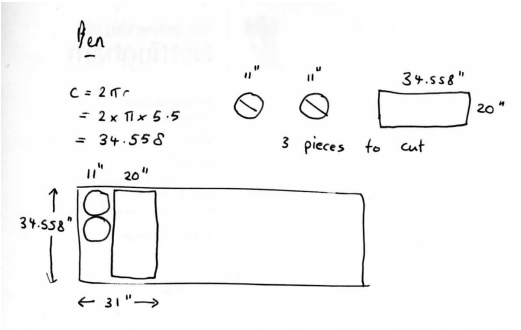
Aisha provides only one solution to Q2, and so does not show that her solution is optimal. She does not compare alternatives.

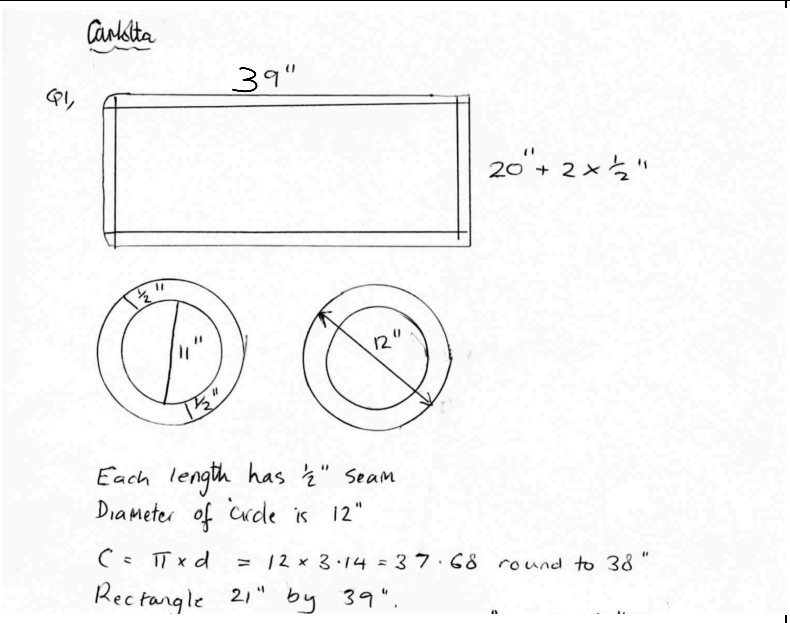
Aisha could improve her solution by taking seam allowances into account, working to an appropriate degree of accuracy, and by considering different options in Q2.

Ben draws minimal diagrams for Question 1. He shows calculations without explaining their relevance to the diagrams. Ben forgets to add seam allowances to the width of the rectangle and the circle. So the measures he uses are too short.

Ben records calculations correct to three decimal places. Even cutting fabric accurate to the nearest tenth of an inch would be difficult! Rounding to the nearest half- or quarter-inch is appropriate in this context.

Ben's solution could be improved if he explained his work more clearly, for example, showing the connections between diagrams and calculations. He needs to take seam allowances into account, and work to an appropriate degree of accuracy. He could also provide alternative responses to Q2, to justify his solution.





Carlotta correctly draws the pattern pieces, remembering to take account of hems. She works to an appropriate degree of accuracy.

She incorrectly adds hems to the diameter of the circle before calculating the length of the rectangle. So her rectangle is so long its length will not fit within the width of the fabric.

Carlotta shows she understands the issue of different arrangements of pattern pieces by giving examples. This is an attempt to justify her solution to Q2 by showing one solution and a non-solution. Her answer is incorrect because she has made the rectangle of fabric too long.

Her solution could be improved by better explanation in Q1, and by excluding the hems from her calculation for the length of the rectangle.

Next lesson: Review individual solutions to *Designing a Sports Bag* (10 minutes)

Ask students to look again at their original individual solutions to the problem.

Read through your original solution to the Designing a Sports Bag problem.

Write what you have learned during the lesson.

Suppose a friend began work on this task tomorrow. What advice would you give your friend to help him or her produce a good solution?

Some teachers set this task as homework.

Solutions

1. The student's sketch should show two circular ends, and a rectangular piece. Each piece should show hems of ½ inch all round.

The circular ends are of diameter 11" with half an inch added all round, to give 12" as the total diameter of the piece cut. However, the length of the rectangular piece is the same as the circumference of the circular end *without* seam allowances:

$C = \pi d = \pi \times 11 = 34.5564... \approx 34.75"$

Alternatively, if *d=11", r=5.5".* Then $C = 2\pi r = 2 \times \pi \times 5.5 = 34.5564... \approx 34.75"$.

Rounding up is required, because the rectangle must be *at least* as long as the circumference in this context. Rounding to the nearest ¼ or ½ inch is appropriate for the context of cutting cloth by hand.

Some students may add ½" or 1" hem before calculating the circumference, giving an incorrect measure for the length of the rectangle. The length of the rectangular piece is the same as the circumference of the circular end *without* seam allowances; that length is sewn to the circumference.

The rectangular piece thus measures:

$(20 + 2 \times \frac{1}{2})$ by $(34.75 + 2 \times \frac{1}{2})$ or 21" by 35.75".

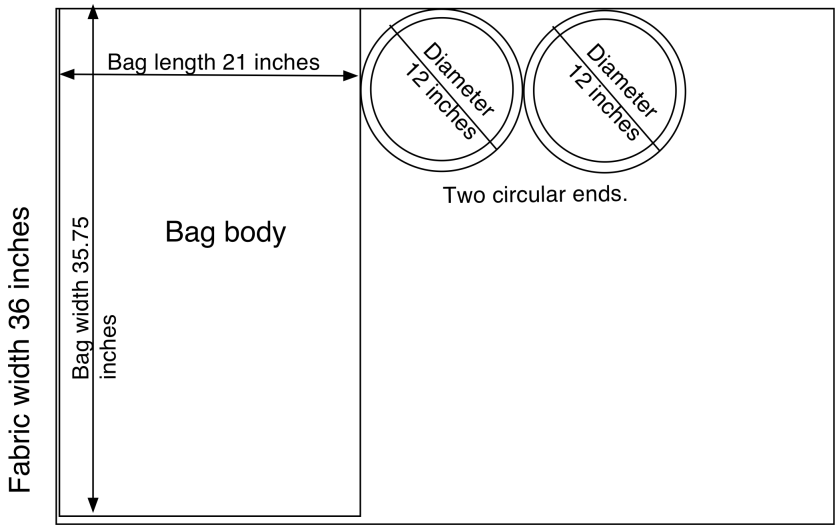
All diagrams should be labeled with the measure of the length, what the length is, and show seam allowances.

Calculations should be recorded, and the reasons for those calculations explained.

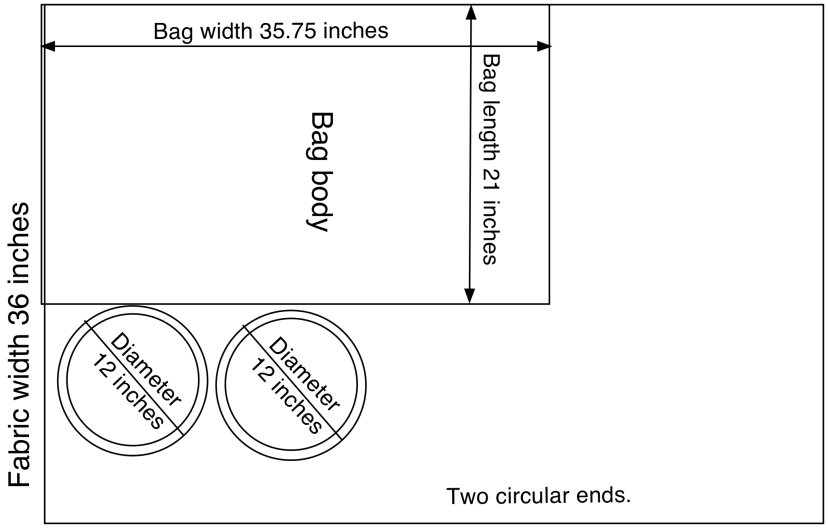
2. The student should draw at least one sketch, showing how the pieces can be arranged. S/he should give a suitable length for the cloth that is required. Ideally, this would include small gaps to ease cutting out.

A full solution will compare some different arrangements, and explain which are optimal. There should be evidence that alternatives have been considered and dismissed. Diagrams should be fully labeled, and calculations explained, as above.

In this orientation, 45 inches of fabric is needed.

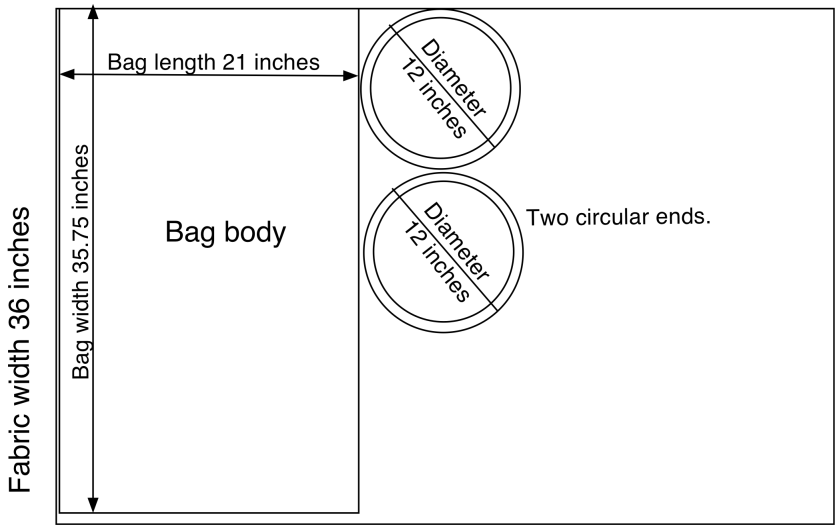


By rotating the bag body through 90 degrees, the circular end pieces fit underneath.



For this orientation, 36 inches of fabric is needed (rounded to the nearest purchasable inch).

Rotating the fabric and putting the circles vertically in line with each other requires only 33 inches of fabric. This seems to be optimal. You might push students to say whether, and how, they can be sure of this result.

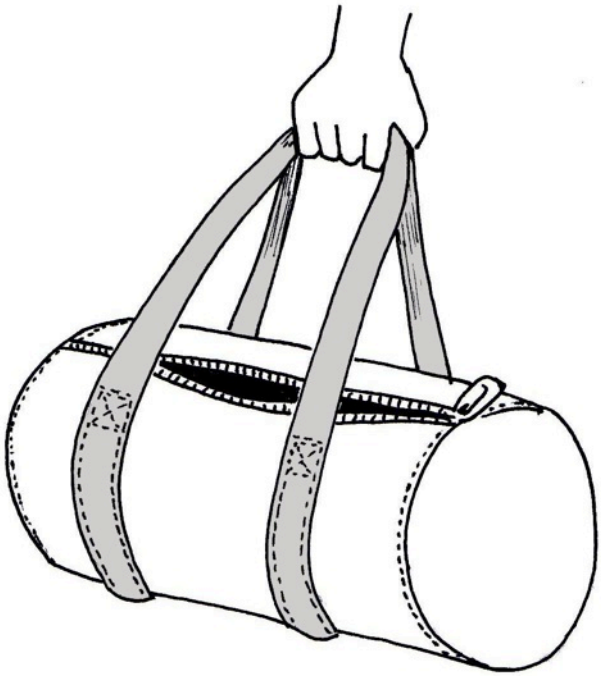


Working systematically through different ways of placing the cloth, identifying congruent arrangements, produces an adequate, exhaustive proof of the result.

Some students might like to figure out whether cutting the fabric for the body of the bag on the cross (at an angle of 45 degrees to the top edge of the bolt) would change the optimal solution.

Designing a Sports Bag

You have been asked to design a sports bag.



- The length of the bag will be 20 inches.
- The bag will have circular ends of diameter 11 inches.
- The main body of the bag will be made from 3 pieces of material: a piece for the curved body, and the two circular end pieces.
- Each piece will need to have an extra ½ inch hem all around it, so that the pieces may be stitched together.

1. Make a sketch of the pieces you will need to cut out for the body of the bag. On your sketch, show all the measurements you will need.

2. Suppose you are going to make one of these bags from a roll of cloth 1 yard wide. What is the shortest length of cloth you could cut from the roll? Describe, using words and sketches, how you arrive at your answer.

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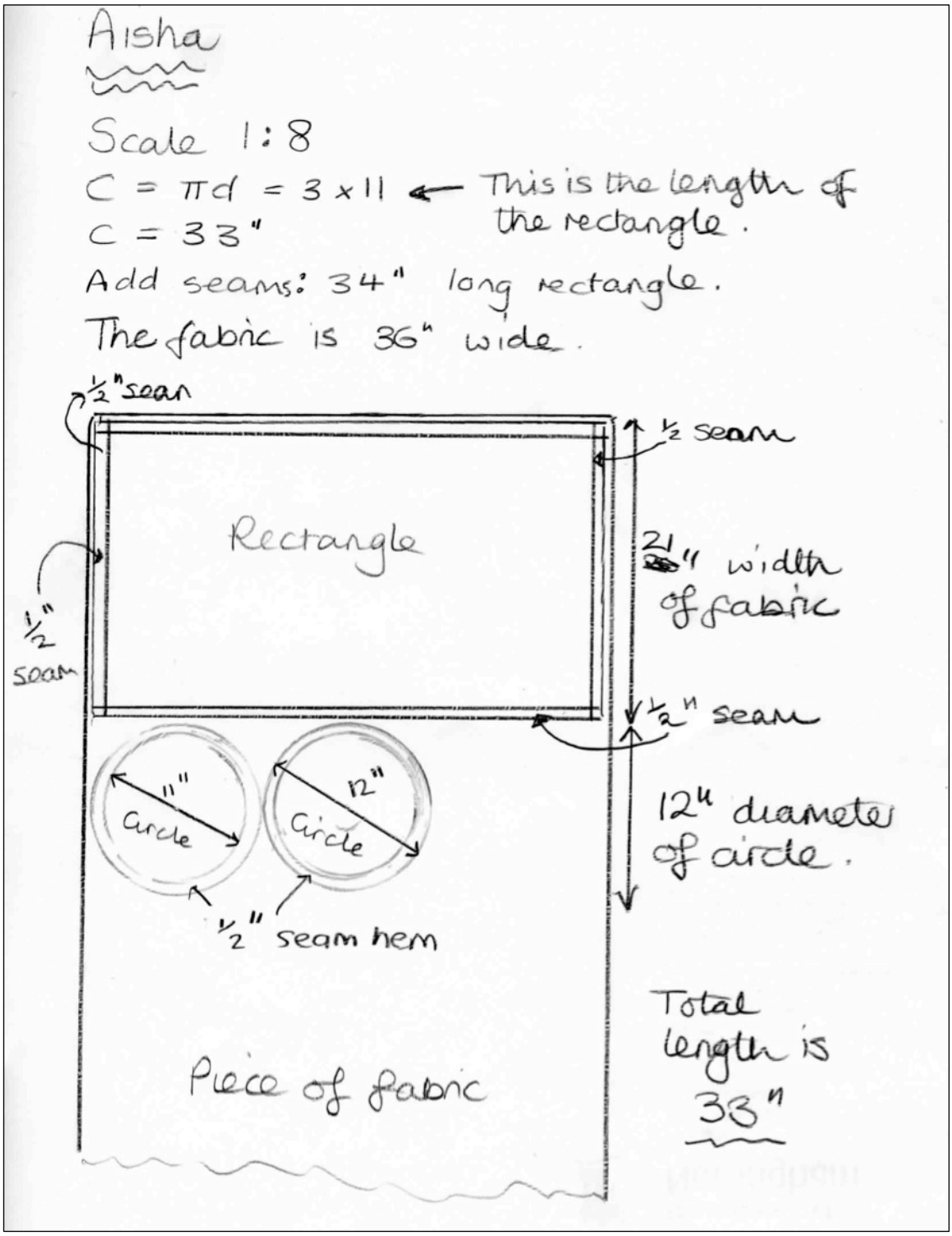
Sample Responses to Discuss: Aisha

Imagine you are Aisha's teacher.

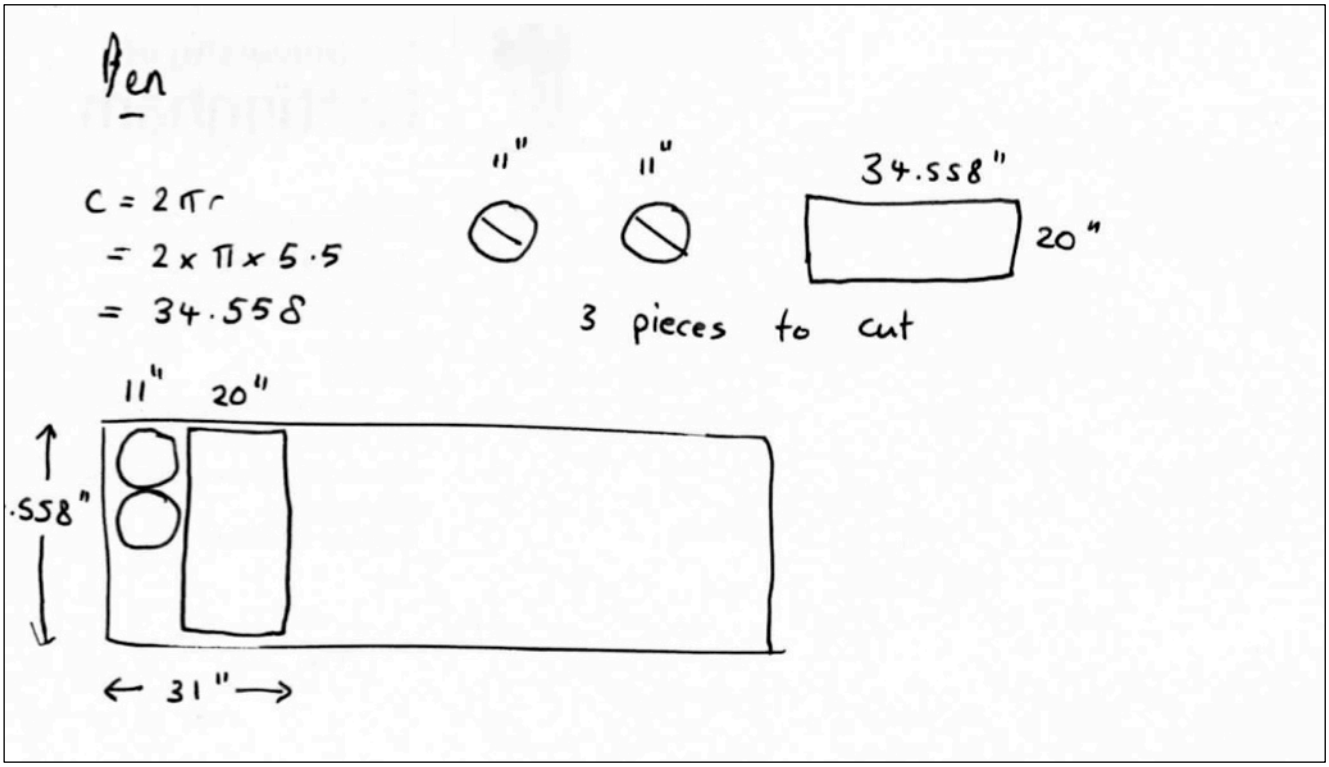
- What do you like and dislike about Aisha's work?
- What is unclear about the work?
- How could Aisha improve her solution?



Sample Responses to Discuss: Ben

Imagine you are Ben's teacher.

- What do you like and dislike about Ben's work?
- What is unclear about the work?
- How could Ben improve his solution?



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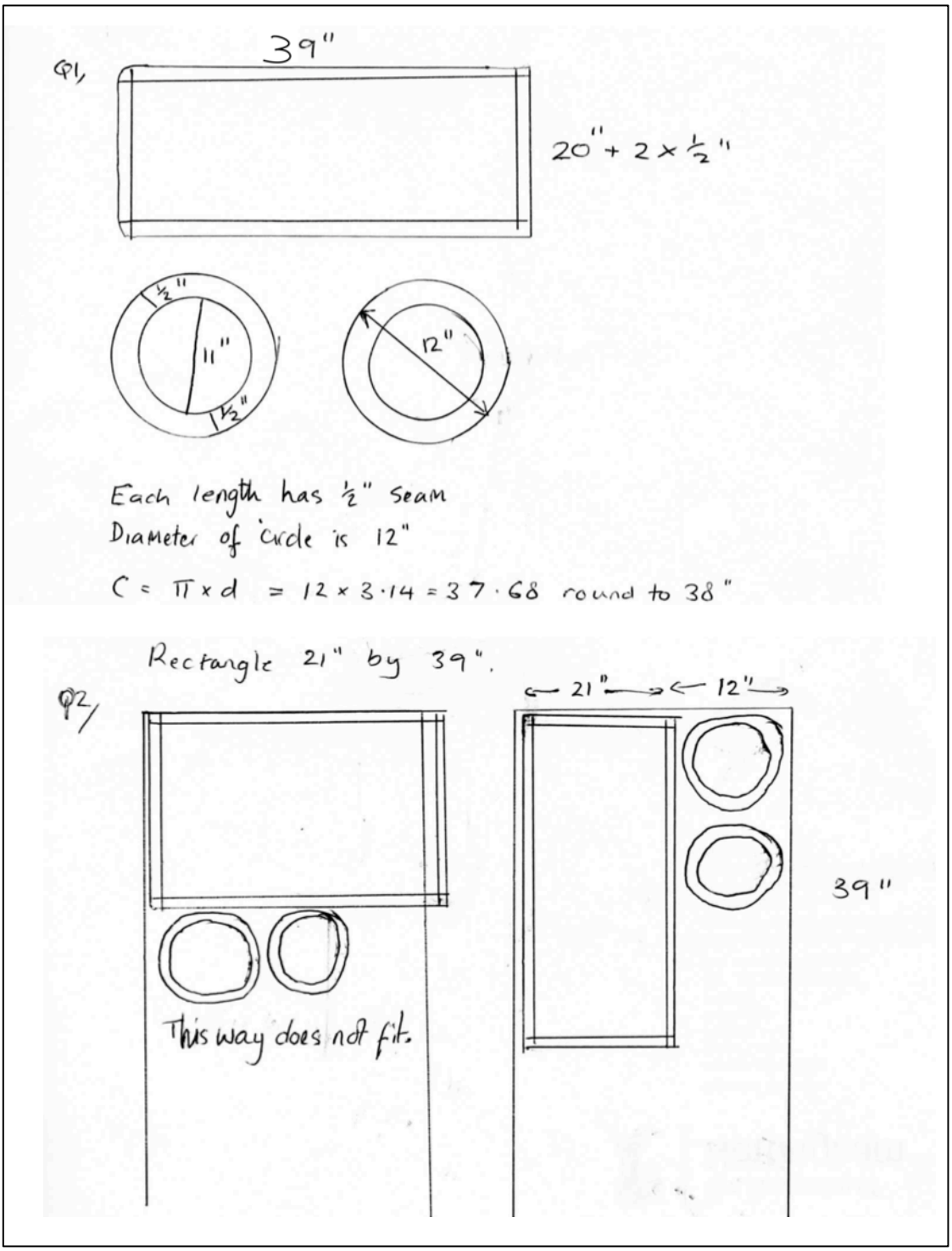
Sample Responses to Discuss: Carlotta

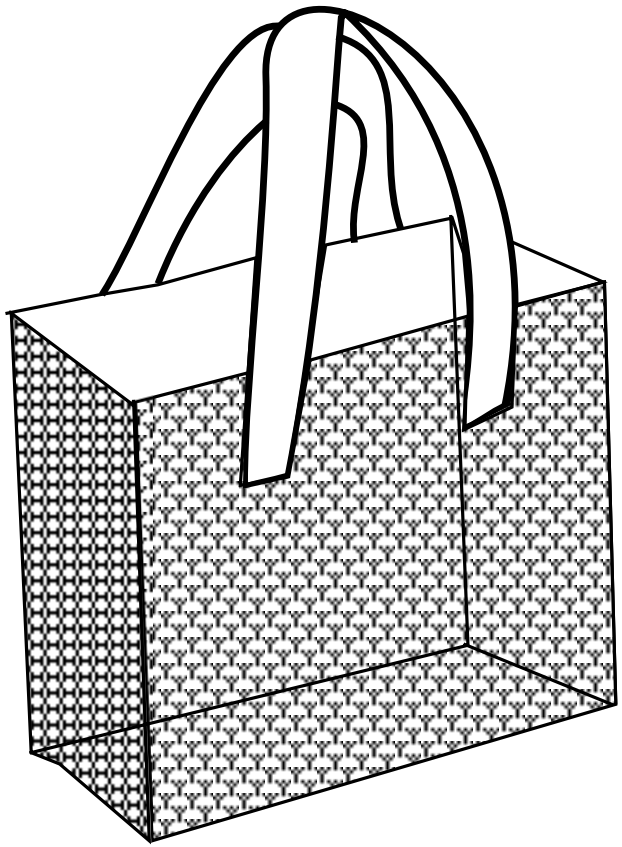
Imagine you are Carlotta's teacher.

- What do you like and dislike about Carlotta's work?
- What is unclear about the work?

How could Carlotta improve her solution?



Making a tote bag



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Unrolling a bolt of fabric

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Placing Paper Pattern Pieces

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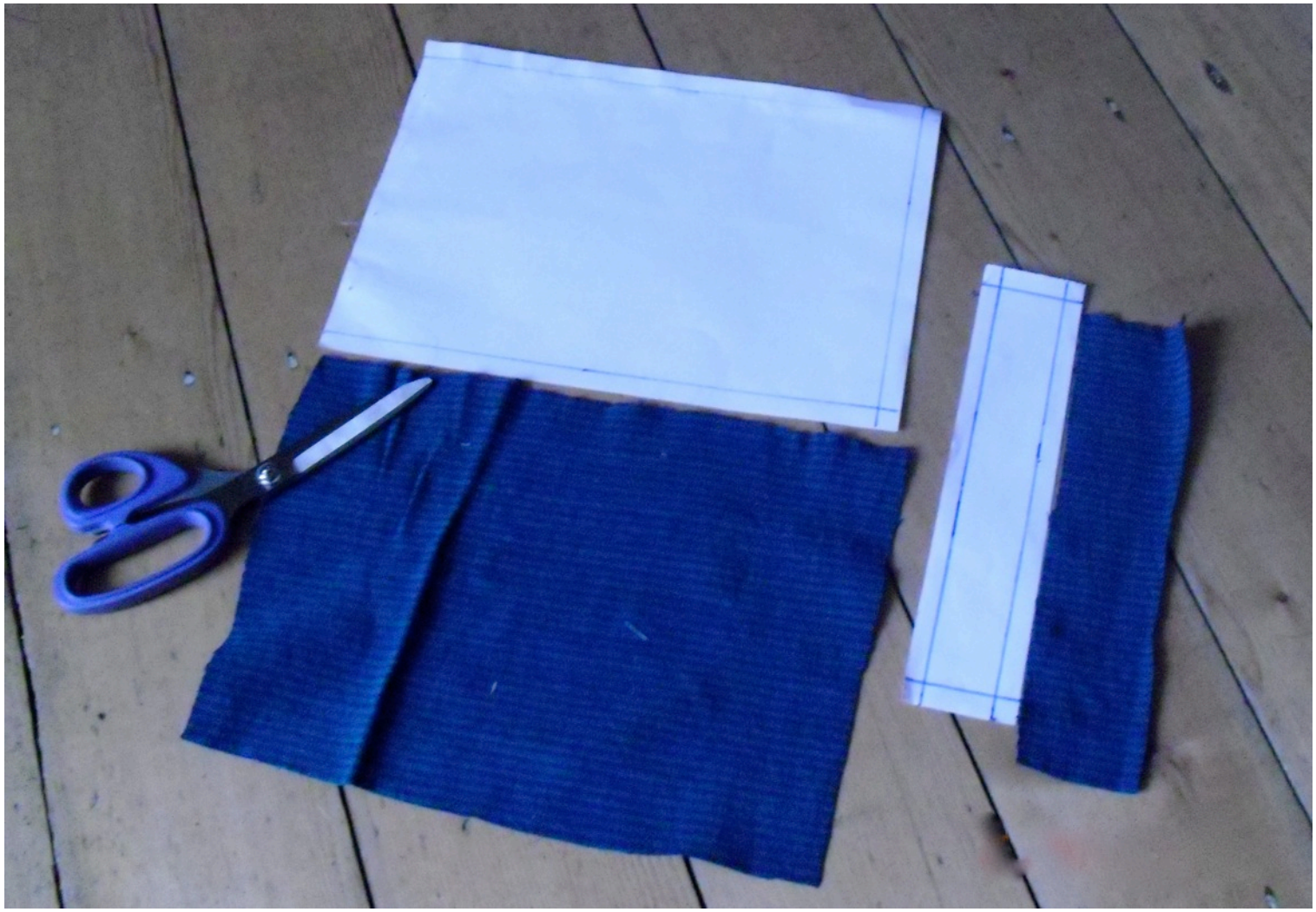
Pinning and Cutting Before Sewing



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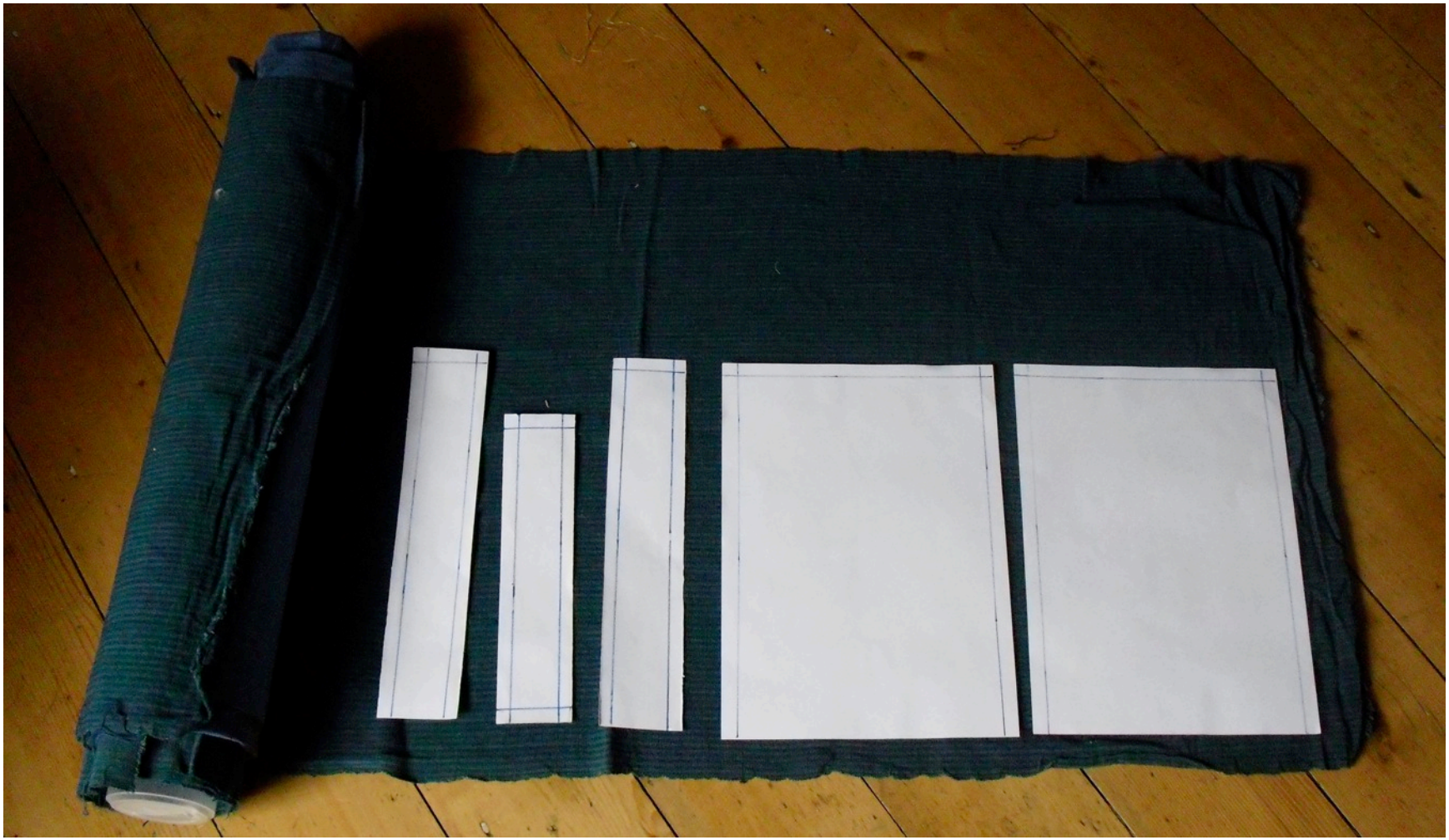
Arrange the Pattern Pieces on the Cloth



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Minimize the Length of Fabric Purchased

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Designing a Sports Bag

- The length of the bag will be 20”.

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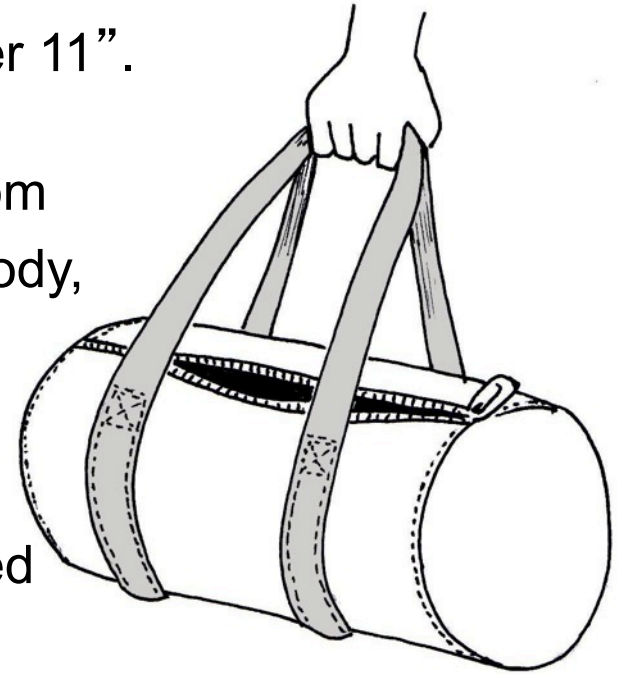
- The main body of the bag will be made from 3 pieces of material: a piece for the curved body, and the two circular end pieces.

- Each piece will need to have an extra ½” all around it so that the pieces may be stitched together. This is the seam allowance or hem.



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Analyzing Sample Responses to Discuss

Work together on one solution at a time.

- **What do you like/dislike about the work?**
- **What is unclear about the work?**
- **What could the student do to improve this solution?**

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Aisha's Work Q1

Aisha

Scale 1:8

$C = \pi d = 3 \times 11$ ← This is the length of the rectangle.

$C = 33''$

Add seams: 34" long rectangle.

The fabric is 36" wide.

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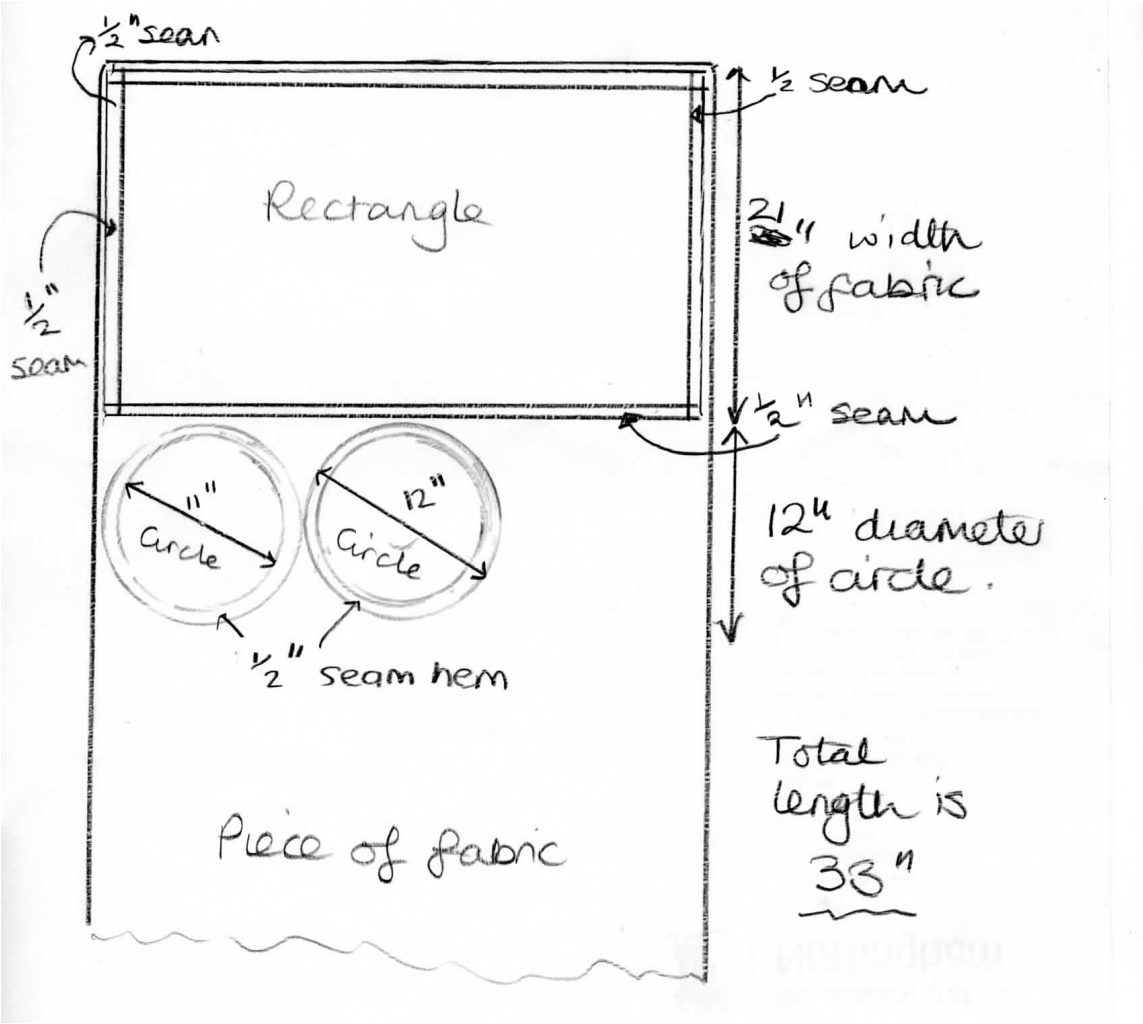
Aisha's Work Q2



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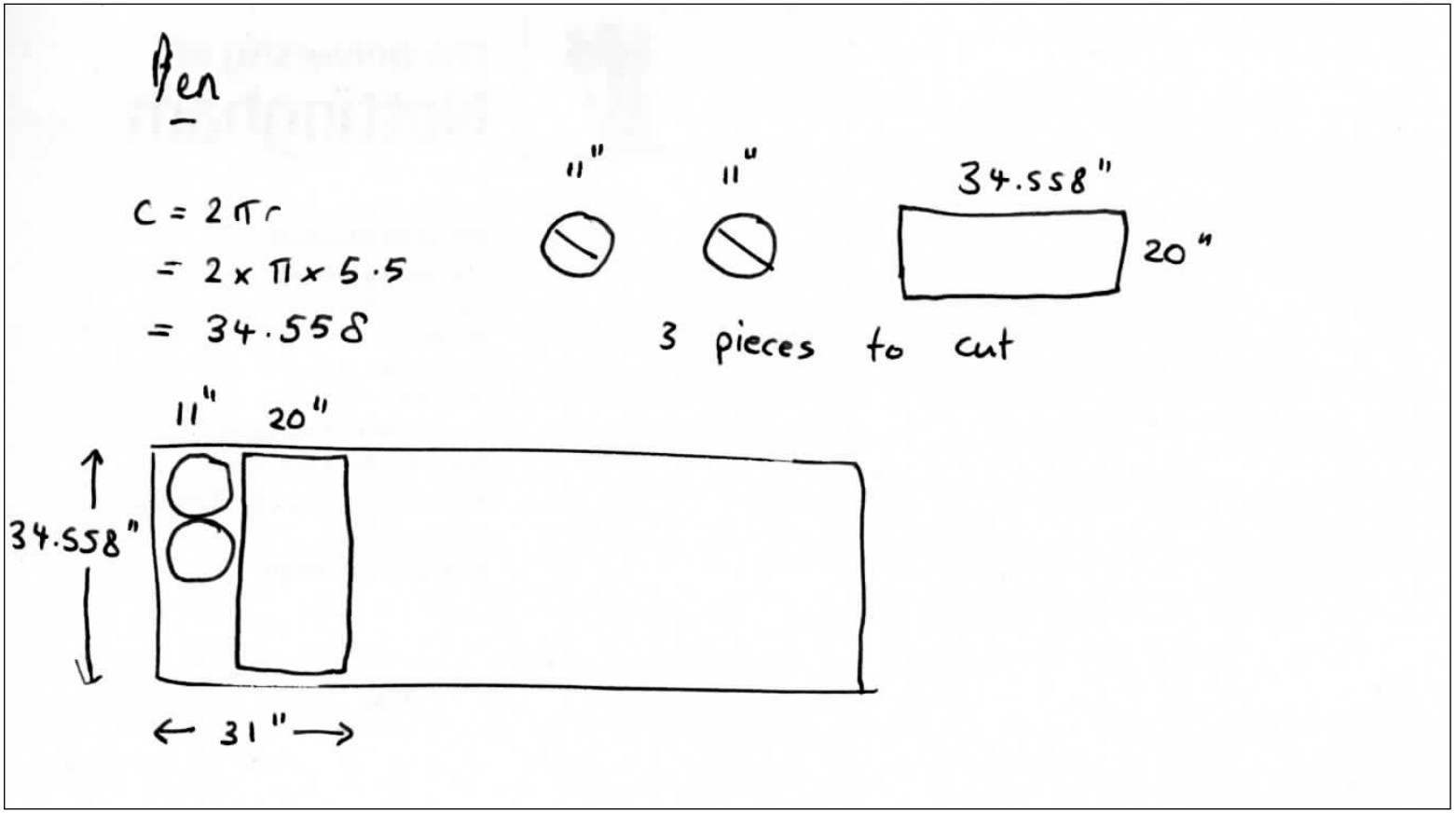
Ben's Work



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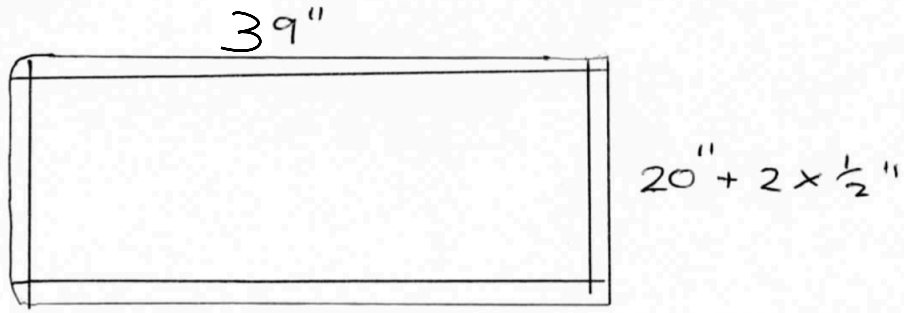
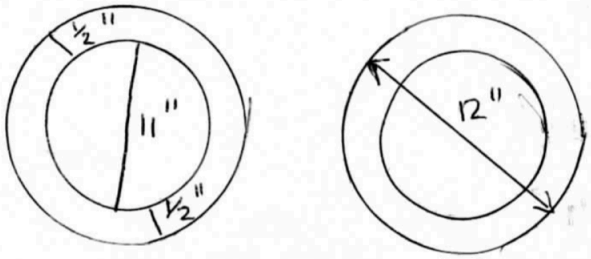
Carlotta's Work Q1

Carlotta

Q1,





Each length has ½" seam

Diameter of circle is 12"

$C = \pi \times d = 12 \times 3.14 = 37.68$ round to 38"

Rectangle 21" by 39".

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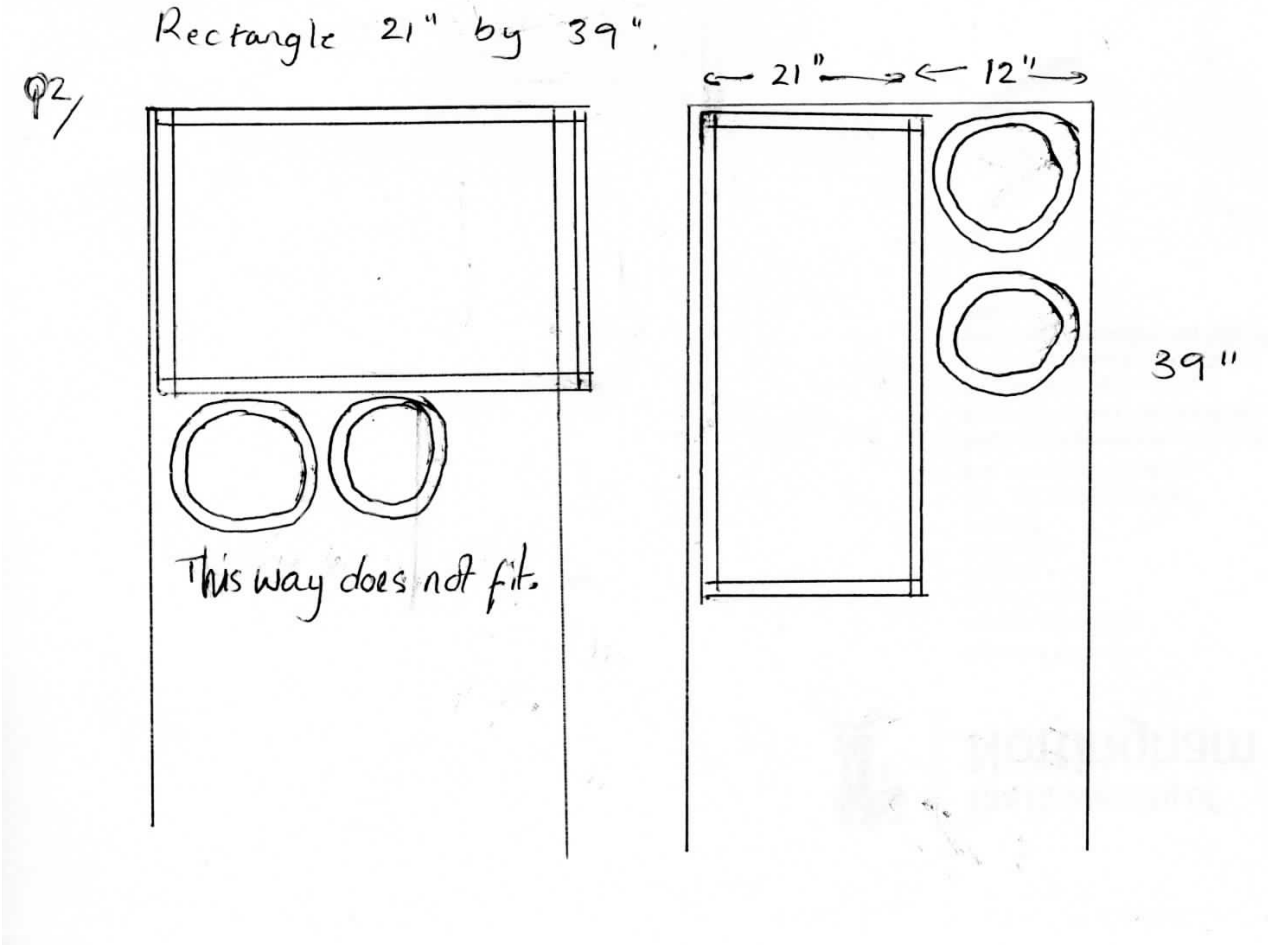
Carlotta's Work Q2



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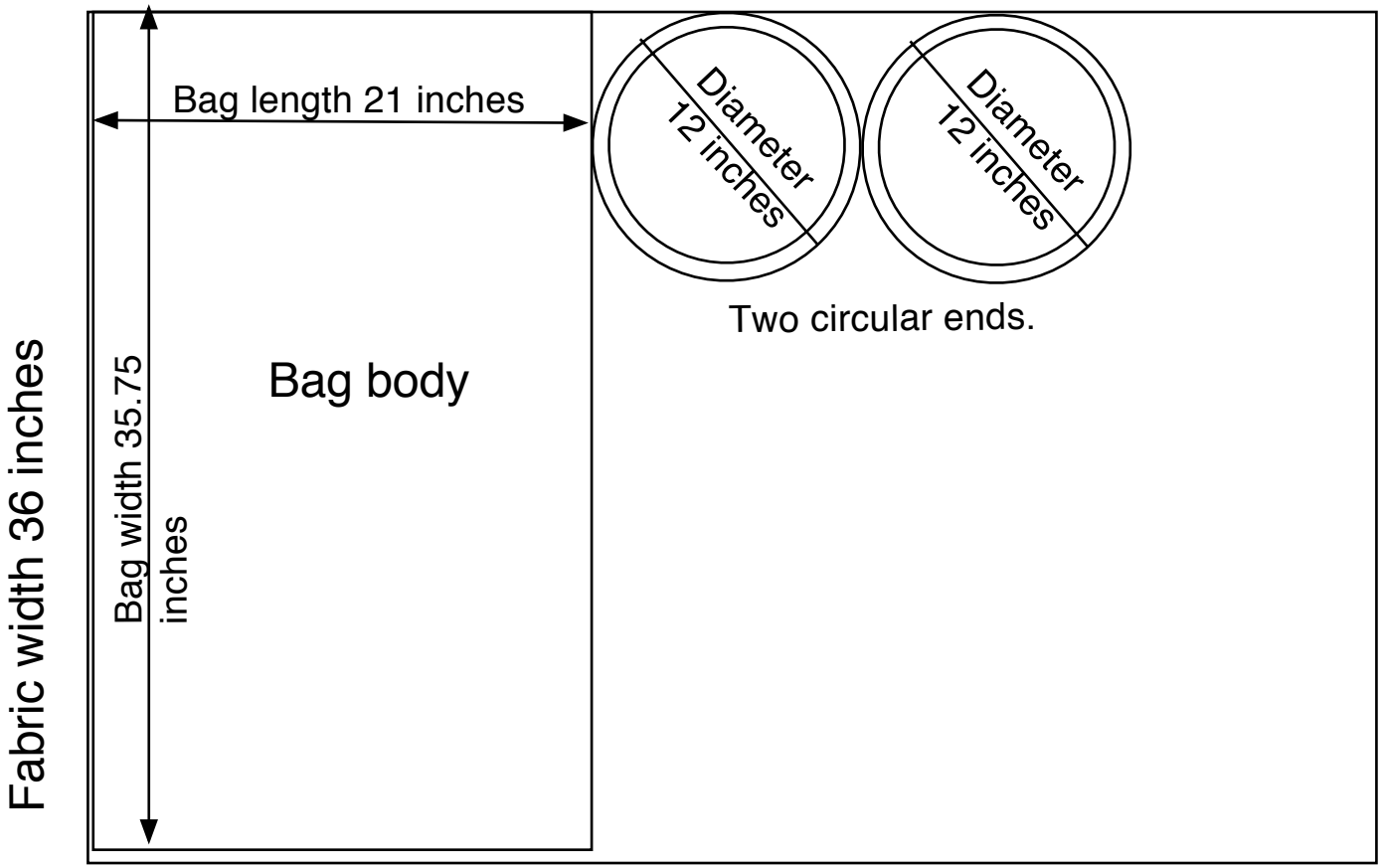
Which Arrangement Uses Least Fabric? (1)



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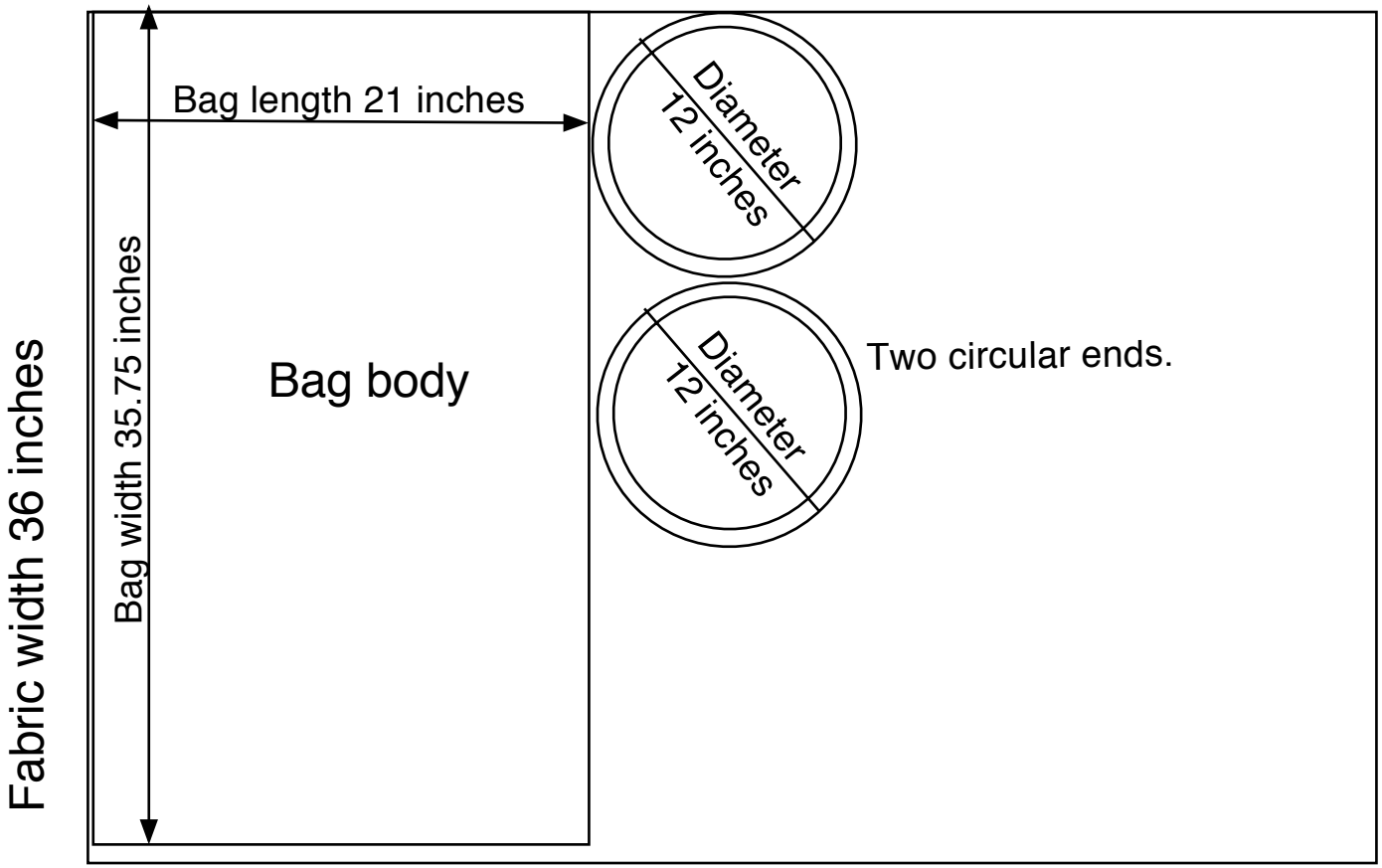
Which Arrangement Uses Least Fabric? (2)



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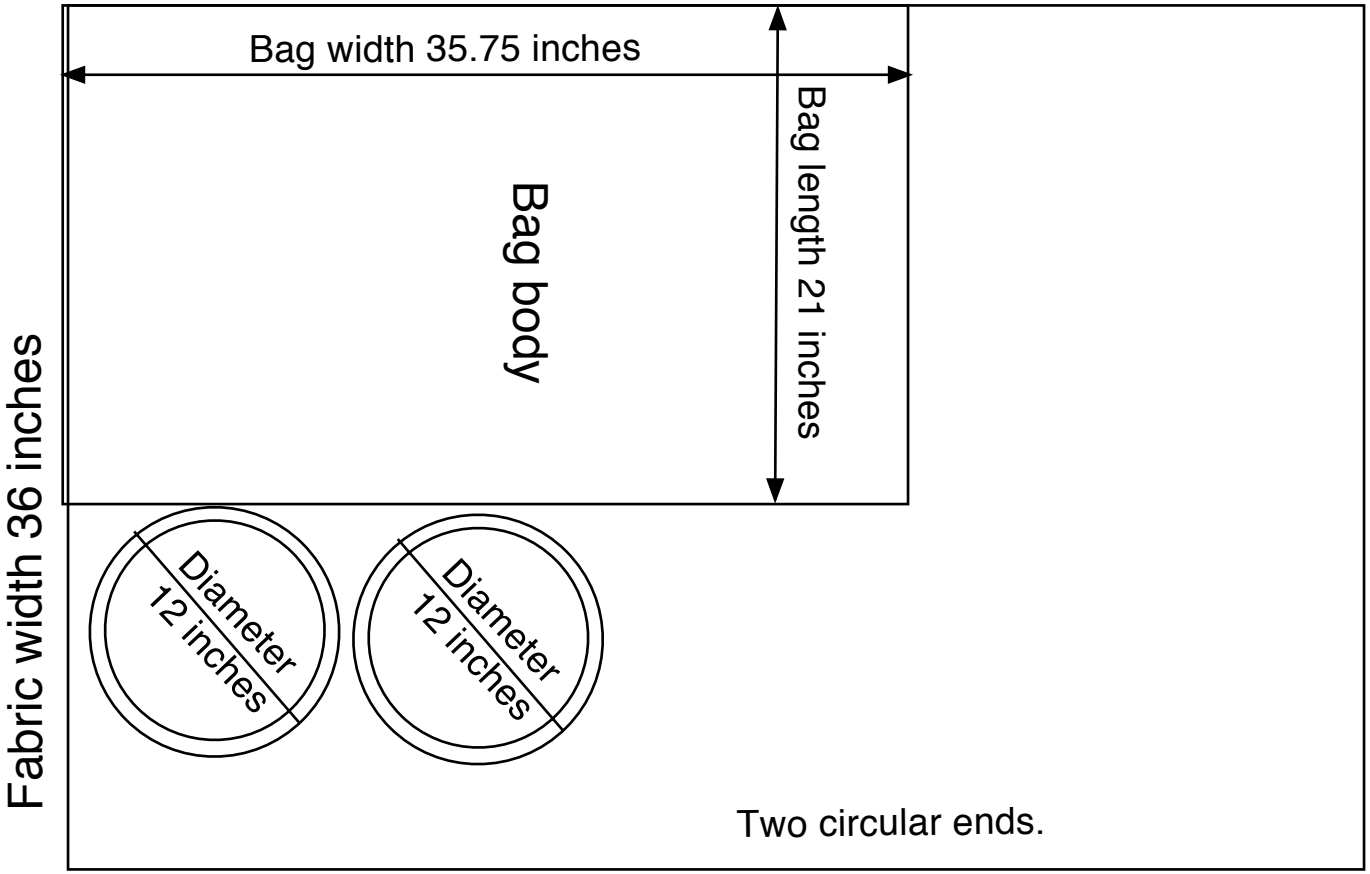
Which Arrangement Uses Least Fabric? (3)



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