Mathematics Assessment Project

Formative Assessment Lesson Materials

Estimating Length Using Scientific Notation

MARS Shell Center
University of Nottingham & UC Berkeley
Alpha Version

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Estimating Length Using Scientific Notation

Mathematical goals

This lesson unit is intended to help you assess how well students are able to:

- Estimate lengths of everyday objects.
- Convert between scientific and decimal notation.
- Make comparisons of the size of numbers expressed in both scientific and decimal notation.

Standards addressed

This lesson involves *mathematical content* in the standards from across the grades, with emphasis on:

8-EE: Work with radicals and integer exponents.

This lesson involves a range of *mathematical practices*, with emphasis on:

2 Reason abstractly and quantitatively.
7. Look for and make use of structure.

Introduction

The unit is structured in the following way:

- Before the lesson, students work individually on an assessment task that is designed to reveal their current understanding and difficulties. You then review their work and create questions for students to consider, in order to improve their solutions.
- During the lesson, students work in small groups on a collaborative task, matching measurements expressed in decimal and scientific notation. They then match these measurements to everyday objects.
- Students then make comparisons between the objects by investigating the size of one object in relation to another.
- After a plenary discussion, students revise their solutions to the assessment task.

Materials required

- Each student will need two copies of the assessment task, *Size It Up*, a mini-whiteboard, a pen, and an eraser.
- Each small group of students will need *Card Set A: Measurements*, *Card Set B: Objects*, *Card Set C: Comparisons* (cut into cards before the lesson), a large sheet of paper, and a glue stick.
- There is a projector resource to support whole-class discussions.

Time needed

Approximately fifteen minutes before the lesson for the assessment task, a seventy-five minute lesson, and ten minutes in a follow-up lesson (or for homework). Timings given are only approximate. Exact timings will depend on the needs of the class.

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Before the lesson

Assessment task*: Size It Up* (15 minutes)

Have the students complete this task, in class or for homework, a day or more before the formative assessment lesson. This will give you an opportunity to assess the work, and to find out the kinds of difficulties students have with it. You will then be able to target your help more effectively in the follow-up lesson.

Give each student a copy of the assessment task *Size It Up*.

Read through the questions and try to answer them as carefully as you can.

It is important that students are allowed to answer the questions without your assistance, as far as possible.

Students should not worry too much if they cannot understand or do everything, because in the next lesson they will work on a similar task, which should help them. Explain to students that by the end of the next lesson, they should be able to answer questions such as these confidently. This is their goal.

Size It Up

1. The table contains seven measurements. Some have been written in scientific notation and some in decimal notation.

   (a) Complete the last two columns of the table so that each measurement is expressed in both scientific and decimal notation.

   (b) In the first column, rank the measurements in size order where rank 1 is the smallest. One has been done for you already.

Rank	Scientific Notation	Decimal Notation
	1×10^{-2} m	
		0.004 m
		200 m
	8×10^{5} m	
		40,000,000 m
4		40 m
	8×10^{-4} m	

2. Choose two measurements from the table to complete the following statements. Use decimal notation in part (a) and scientific notation in part (b):

   (a)

   ..... × 50 = ...

   (b)

   ..... × 4000 = ...

Students who sit together often produce similar answers, and then when they come to work together on similar tasks, they have little to discuss. For this reason, we suggest that when students do the assessment task, you ask them to move to different seats. Then at the beginning of the formative assessment lesson, allow them to return to their usual places. Experience has shown that this produces more profitable discussions.

Assessing students' responses

Collect students' responses to the task, and note down what their work reveals about their current levels of understanding and their different approaches.

We suggest that you do not score students' work. The research shows that this will be counterproductive, as it will encourage students to compare their scores, and will distract their attention from what they can do to improve their mathematics.

Instead, help students to make further progress by summarizing their difficulties as a series of questions. Some suggestions for these are given on the next page. These have been drawn from common difficulties observed in trials of this unit.

We suggest that you write a list of your own questions, based on your students' work, using the ideas that follow. You may choose to write questions on each student's work. If you do not have time to do this, select a few questions that will be of help to the majority of students. These can be written on the board at the end of the lesson.

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Common issues:	Suggested questions and prompts:
Converts measurements from scientific to decimal notation incorrectly For example: Writes 1×10^{-2} m as 0.001 m.	• *What does $\times 10^{-1}$ mean? What does $\times 10^{-2}$ mean?* • *How can you check your answer is correct?*
Does not understand the requirements for scientific notation For example: Writes 40,000,000 m as 40×10^{6} m.	• *Have any of these numbers been written in scientific notation?* *20×10^{-2}, 0.6×10^{4}, 5×10^{3}* *How do you know?*
Does not fully understand the effects of negative exponents For example: The student assumes that 8×10^{-4} is bigger than 1×10^{-2}.	• *Is 10^{-4} bigger than 10^{-2}? How do you know?* • *In scientific notation, which has a bigger effect on the size of a number, the coefficient or the exponent? Explain.* • *Is 9×10^{1} bigger than 1×10^{2}? What is the difference between the two numbers?* • *Is 9×10^{-2} bigger than 1×10^{-1}? What is the difference between the two numbers?*
Fails to complete the size rankings for the measurements	• *Which measurement is the smallest? Which measurements is the biggest? How do you know?*
Makes incorrect assumptions when comparing measurements For example: The student assumes that Q.2 cannot be completed as the coefficients in the scientific notation are limited to 1, 2, 4 and 8 and so you can't multiply any of these by either 50 or 4000 to get another listed coefficient.	• *Can you use the decimal notation to help you to complete the second question?* • *Numbers in scientific notation include an integer power of ten. What effect does this have?*
Completes the task The student needs an extension task.	• *Choose two measurements that you have not yet used in Q.2. Can you work out how much bigger one measurement is than the other?* • *How much bigger is the measurement you have ranked as 2 than rank 1? Can you compare your other rankings in this way?*

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Suggested lesson outline

Whole-class introduction (20 minutes)

Give each student a mini-whiteboard, pen and eraser.

Display slide 1 of the projector resource:

What's the Number?

$30{,}000 \times 10^{-1}$

3×10^{3}

0.3×10^{4}

These three expressions all represent the same number. Write the number on your whiteboard [3000.]

It is likely that students will use the second representation to figure out the number. Once the correct number has been agreed upon, ask students to explain which expression they used and why. Students should be confident in checking that all three expressions represent the same number.

Display slide 2 of the projector resource:

Scientific Notation or Not?

$30{,}000 \times 10^{-1}$

3×10^{3}

0.3×10^{4}

Which of these three expressions is written in scientific notation? Write on your whiteboard any expressions that you think have been written using scientific notation.

Some students may assume that all three expressions are in scientific notation. Spend some time discussing student responses and address any misconceptions that arise. Students should have a clear understanding that 3×10^{3} is the only expression written using scientific notation and be able to justify why the other two representations are not scientific notation.

Display slide 3 of the projector resource:

Which is bigger?

8×10^{-3}

4×10^{-1}

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Write on your whiteboard the number you think is the bigger of the two.

If students' opinions are divided, ask students to pair up with another student with the same choice.

In your pair, work out how much bigger your chosen number is than the other number.

Once students have had a chance to complete this calculation, choose a few pairs of students to feedback their choice and any calculations they have carried out. A common misconception is to ignore the exponent and assume that 8×10^{-3} is twice as big as 4×10^{-1} due to the fact that $4 \times 2 = 8$. If all students have correctly identified the bigger of the two numbers, it may be appropriate to explore the different methods used to compare the two numbers and address any differences in the comparison of sizes.

Collaborative activity: *Card Set A: Measurements* (15 minutes)

Organize the class into groups of two or three students.

Give each group the *Card Set A: Measurements*.

Explain how students are to work collaboratively.

Take it in turns to match a measurement card in scientific notation with a measurement card in decimal notation.

Each time you match a pair of cards, explain your thinking clearly and carefully. Place your cards side by side on your desk, not on top of one another, so that everyone can see them.

Partners should either agree with the explanation, or challenge it if it is not clear, or not complete.

It is important that everyone in the group understands the matching of each card.

You should find that some cards cannot be paired. Write the alternative form of these measurements on the blank cards.

Slide 4 of the projector resource summarizes these instructions.

While students work in small groups you have two tasks, to make a note of student approaches to the task, and to support student reasoning.

Make a note of student approaches to the task

Listen and watch students carefully. In particular, notice how students make a start on the task, where they get stuck, and how they overcome any difficulties. Do students prefer to convert from scientific notation to decimal notation or the other way around? How are they working out the value of the exponent? You can use this information to focus a whole-class discussion towards the end of the lesson.

Support student reasoning

Try not to make suggestions that move students towards a particular matching. Instead, ask questions to help students to reason together. You may want to use some of the questions in the *Common Issues* table. If you find one student within the group has matched a particular pair of cards, challenge another student in the group to explain the match.

Chen matched these two cards. Christie, why does Chen think these two cards match?

When completing the blank measurement cards, some students may assume that for negative exponents, the magnitude of the exponent corresponds to the number of zeros after the decimal point (for example, assuming that $1.2 \times 10^{-2} = 0.0012$ rather than 0.012.) Encourage students to explain their reasoning and use this process as a method for checking their work.

Collaborative activity: *Card Set B: Objects* (15 minutes)

When students have finished matching the measurement cards, give each group *Card Set B: Objects*.

Students may already have noticed that each measurement card has an "m" at the end. Ask students to suggest what this represents and clarify that all of the measurements are in meters.

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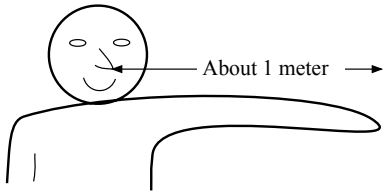
For the next part of the lesson students will be matching up an everyday object from *Card Set B* with the appropriate measurement cards from *Card Set A*. It might be helpful to illustrate the rough size of a meter. Explain to students that the distance from your nose to a fingertip is about one meter.



Ask learners to match the objects *Card Set B* with the corresponding measurements *Card Set A*.

If students are struggling, suggest putting the object cards in order of size first and then matching them up to the measurements cards based on order of size.

As students complete all matches, ask them to check their work against that of a neighboring group.

> *If there are differences between your matching, ask for an explanation. If you still don't agree, explain your own thinking.*

Collaborative activity: *Card Set C: Comparisons* (15 minutes)

When students are satisfied with their matches, give each group *Card Set C: Comparisons*, a glue stick, and a large sheet of paper for making a poster:

> *You are now to place an arrow card between two sets of measurements/objects to show how much bigger one object is than another. You can then stick these cards down on your poster. Do this for as many pairs as possible. There are some blank arrow cards for you to use if needed.*

As students work on the task, encourage them to check their work. They may find it helpful to think about the matched objects to check whether or not their comparison is realistic, before carrying out any calculations:

> *Roughly how many times bigger is the height of a tree than the height of a door? Is this a realistic comparison? Can you now check it mathematically?*

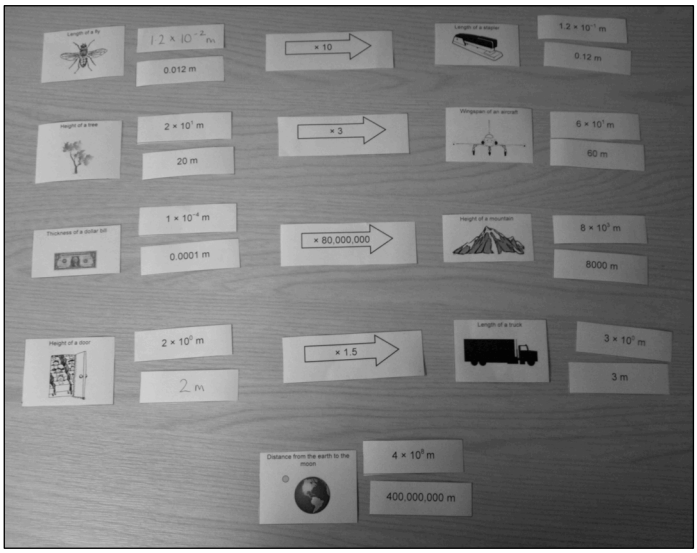
Students should be encouraged to justify the comparisons made and describe their methods:

> *Can you explain why you placed this arrow here?*

> *In this task, do you find it easier to use the decimal notation card or the scientific notation card? Why?*

When students have completed as many comparisons as possible, ask them to stick down any remaining matched up cards on their poster.

Their finished poster may look something like this:



Extension activity

If a group successfully completes four comparisons, ask them to compare their remaining object with an object of their own choice. They need to decide on an object and it's measurement and state how many times bigger it is.

Whole class discussion (10 minutes)

Organize a discussion about what has been learned and encourage students to explore the different methods used.

Michael, explain how you matched one arrow card with two measurement cards.

In addition to asking for a variety of methods, pursue the theme of listening and comprehending others' methods by asking students to rephrase others' reasoning.

Can someone else put that into your own words?

Does anyone disagree with this pairing?

It is likely that each poster will be slightly different, dependent on the comparisons made. There will not be time to discuss all the pairings, but a good coverage of measurements should be encouraged.

Revisiting the initial assessment task (10 minutes)

Return to the students their original assessment *Size It Up*, as well as a second blank copy of the task.

Look at your original responses and think about what you have learned this lesson. Using what you have learned, try to improve your work.

If you have not added questions to individual pieces of work, then write your list of questions on the board. Students should select from this list only the questions they think are appropriate to their own work.

If you find you are running out of time, then you could set this task in the next lesson or for homework.

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Solutions

Assessment Task: ***Size It Up***

Rank	Scientific Notation	Decimal Notation
3	1×10^{-2} m	0.01 m
2	4×10^{-3} m	0.004 m
5	2×10^{2} m	200 m
6	8×10^{5} m	800,000 m
7	4×10^{7} m	40,000,000 m
4	4×10^{1} m	40 m
1	8×10^{-4} m	0.0008 m

a.

$$40\,\text{m} \times 50 = 200\,\text{m}$$

or

$$800{,}000\,\text{m} \times 50 = 40{,}000{,}000\,\text{m}$$

b.

$$1 \times 10^{-2}\,\text{m} \times 4000 = 4 \times 10^{1}\,\text{m}$$

or

$$2 \times 10^{2}\,\text{m} \times 4000 = 8 \times 10^{5}\,\text{m}$$

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Collaborative activity

Scientific Notation	Decimal Notation	Object
1×10^{-4} m	0.0001 m	Thickness of a dollar bill
1.2×10^{-2} m	0.012 m	Length of a fly
1.2×10^{-1} m	0.12 m	Length of a stapler
2×10^{0} m	**2 m**	Height of a door
3×10^{0} m	3 m	Length of a truck
2×10^{1} m	20 m	Height of a tree
6×10^{1} m	60 m	Wingspan of an aircraft
8×10^{3} m	8000 m	Height of a mountain
4×10^{8} m	400,000,000 m	Distance from the earth to the moon

The measurements in **bold** correspond to blank cards.

There are numerous comparisons that can be made. For example:

Length of a truck 3×10^{0} m = 3 m $\xrightarrow{\times 20}$ Wingspan of an aircraft 6×10^{1} m = 60 m

Thickness of a dollar bill 1×10^{-4} m = 0.0001 m $\xrightarrow{\times 80{,}000{,}000}$ Height of a mountain 8×10^{3} m = 8000 m

Some of the arrows may be redundant depending on the comparisons made.

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Size It Up

1. The table contains seven measurements. Some have been written in scientific notation and some in decimal notation.

   (a) Complete the last two columns of the table so that each measurement is expressed in both scientific and decimal notation.

   (b) In the first column, rank the measurements in size order where rank 1 is the smallest. One has been done for you already.

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4		40 m
	8×10^{-4} m	

2. Choose two measurements from the table to complete the following statements. Use decimal notation in part (a) and scientific notation in part (b):

   (a)

   ..... × 50 = ...

   (b)

   ..... × 4000 = ...

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Card Set A: Measurements

20 m	60 m	0.012 m
0.12 m	3 m	8000 m
400,000,000 m	0.0001 m	
2×10^{0} m	1×10^{-4} m	8×10^{3} m
3×10^{0} m	2×10^{1} m	4×10^{8} m
6×10^{1} m	1.2×10^{-1} m	

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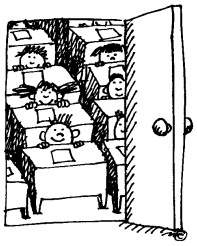
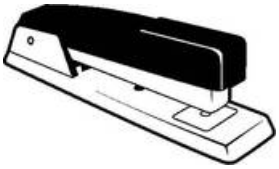
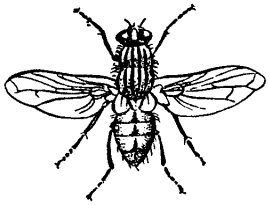
Card Set B: Objects

Height of a tree	Distance from the earth to the moon	Height of a door 
Height of a mountain	Thickness of a dollar bill	Length of a stapler 
Length of a fly 	Length of a truck	Wingspan of an aircraft

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Card Set C: Comparisons

× 1200	× 1.5
× 20,000	× 10
× 20	× 400
× 3	× 80,000,000
× 4000	× 120
× 30	× 30,000

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What's the Number?

$$30{,}000 \times 10^{-1}$$

$$3 \times 10^{3}$$

$$0.3 \times 10^{4}$$

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Scientific Notation or Not?

$$30{,}000 \times 10^{-1}$$

$$3 \times 10^{3}$$

$$0.3 \times 10^{4}$$

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Which is bigger?

$$8 \times 10^{-3}$$

$$4 \times 10^{-1}$$

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Matching Cards

1. Take it in turns to match a measurement card in scientific notation with a measurement card in decimal notation.

2. Each time you match a pair of cards explain your thinking clearly and carefully. Place your cards side by side on your desk, not on top of one another, so that everyone can see them.

3. Partners should either agree with the explanation, or challenge it if it is not clear or not complete.

4. It is important that everyone in the group understands the matching of each card.

5. You should find that some cards cannot be paired. Write the alternative form of these measurements on the blank cards.

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